PREDICTION OF DYNAMIC LATERAL STABILITY CHARACTERISTICS

By

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CERTIFICATE

Certified that this work entitled 'PREDICTION OF DYNAMIC LATERAL STABILITY CHARACTERISTICS' has been carried out under my supervision and that this has not been submitted elsewhere for a degree.

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m	:	Mass of airplane, slugs.
S	:	Wing area, Sq.ft.
$\bar{c}$	:	Wing mean chord, ft.
l	:	Tail length (distance from centre of pressure of vertical tail to centre of gravity, measured parallel to longitudinal stability axis; values of l must be calculated for each angle of attack), ft.
b	:	Wing span, ft.
z	:	Height of centre of pressure of vertical tail above longitudinal stability axis; values of z must be calculated for each angle of attack, ft.
z_W	:	Vertical distance of quarter chord of wing root chord from fuselage centre line, positive downward, ft.
s	:	Non-dimensional time parameter based on span (Vt/b).
A	:	Aspect ratio.
t	:	Time, sec.
V	:	Airspeed, ft. per sec.
k_{X_O}	:	Radius of gyration about principal longitudinal axis of inertia, ft.

k_{Z_o}	:	Radius of gyration about principal normal axis of inertia, ft.
k_X	:	Radius of gyration about X-axis, ft. $\left(\sqrt{k_{X_o}^2 \operatorname{Cos}^2\eta + k_{Z_o}^2 \operatorname{Sin}^2\eta}\right)$.
k_Z	:	Radius of gyration about Z-axis, ft. $\left(\sqrt{(k_{Z_o}^2 \operatorname{Cos}^2\eta + k_{X_o}^2 \operatorname{Sin}^2\eta})\right)$.
K_{X_o}	=	k_{X_o}/b
K_{Z_o}	=	k_{Z_o}/b
K_X	=	k_X/b
K_Z	=	k_Z/b
k_{XZ}	:	Product of inertia factor $((k_{Z_o}^2 - k_{X_o}^2) \operatorname{Sin}\eta \operatorname{Cos}\eta)$.
K_{XZ}	=	k_{XZ}/b^2
K_1	=	K_{XZ}/K_X^2
K_2	=	K_{XZ}/K_Z^2
η	:	Angle of attack of principal longitudinal axis of inertia, deg.
γ	:	Angle of climb or flight path angle, deg.

α	:	Angle of attack of longitudinal body axis, deg.
ϵ	:	Angle between principal longitudinal axis of inertia and longitudinal body axis, deg.
ρ	:	Air density, slugs per ft.^{3}
ϕ	:	Angle of bank, rad.
ψ	:	Angle of yaw, rad.
β	:	Angle of side-slip, rad.
β	:	$\sqrt{1-M^2}$
p	:	Rolling velocity, rad. per sec. $(\frac{d\phi}{dt})$.
r	:	Yawing velocity, rad. per sec. $(\frac{d\psi}{dt})$.
ϕ_o	:	Initial angle of bank, rad.
ψ_o	:	Initial angle of yaw, rad.
β_o	:	Initial angle of sideslip, rad.
$(D\phi)_o$	:	Non-dimensional, initial rolling velocity, $(\frac{d\phi}{d\sigma})$.
$(D\psi)_o$	:	Non-dimensional, initial yawing velocity, $(\frac{d\psi}{d\sigma})$.
R	:	Real part of R+Ii.
I	:	Imaginary part of R+Ii.
A,B,C,D,E	:	Coefficients of the characteristic biquadratic equation.

$P_1, P_2 \ldots, P_7$	:	Factors of the B, C and D coefficients.
$\lambda_1, \lambda_2, \lambda_3, \lambda_4$	:	Roots of the characteristic equation.
D	:	Operator $d/d\sigma$
P	:	Period of lateral oscillations, sec.
$T_{1/2}$	:	Time to damp to one-half amplitude, sec.
τ	:	Time convertion factor $(m/\rho SV)$.
σ	:	Non-dimensional time factor (t/τ).
μ	:	Relative density factor $(m/\rho Sb)$.
L_c	:	Impressed rolling moment, ft.lbs.
N_c	:	Impressed yawing moment, ft.lbs.
Y_c	:	Impressed lateral force, lbs.
C_{l_c}	:	Impressed rolling-moment coefficient.
C_{n_c}	:	Impressed yawing-moment coefficient.
C_{Y_c}	:	Impressed lateral-force coefficient.
C_L	:	Lift coefficient (Lift/qS).
C_D	:	Drag coefficient (Drag/qS).
C_l	:	Rolling-moment coefficient (Rolling moment/qSb).
C_n	:	Yawing-moment coefficient (Yawing moment/qSb).
C_Y	:	Lateral-force coefficient (Lateral force/qS).
q	:	Dynamic pressure lb./sq.ft. $(1/2\, \rho V^2)$.

$$C_{l_\beta} = \partial C_l / \partial \beta$$

$$C_{n_\beta} = \partial C_n / \partial \beta$$

$$C_{Y_\beta} = \partial C_Y / \partial \beta$$

$$C_{l_p} = \partial C_l / \partial \left(\frac{pb}{2V}\right)$$

$$C_{n_p} = \partial C_n / \partial \left(\frac{pb}{2V}\right)$$

$$C_{Y_p} = \partial C_Y / \partial \left(\frac{pb}{2V}\right)$$

$$C_{l_r} = \partial C_l / \partial \left(\frac{rb}{2V}\right)$$

$$C_{n_r} = \partial C_n / \partial \left(\frac{rb}{2V}\right)$$

$$C_{Y_r} = \partial C_Y / \partial \left(\frac{rb}{2V}\right)$$

$$l_\beta = \mu C_{l_\beta} / 2K_X^2$$

$$n_\beta = \mu C_{n_\beta} / 2K_Z^2$$

$$y_\beta = C_{Y_\beta} / 2$$

$$l_p = C_{l_p} / 4K_X^2$$

$$n_p = C_{n_p} / 4K_Z^2$$

$$y_p = C_{Y_p} / 4\mu$$

$l_r \quad = \quad C_{l_r}/4K_X^2$

$n_r \quad = \quad C_{n_r}/4K_Z^2$

$y_r \quad = \quad C_{Y_r}/4\mu$

$l_c \quad = \quad \mu C_{l_c}/2K_X^2$

$n_c \quad = \quad \mu C_{n_c}/2K_Z^2$

$y_c \quad = \quad C_{Y_c}/2$

l_V : Distance of vertical tail M.A.C. from centre of gravity measured along body centre line (body axis), ft.

h : Maximum fuselage height, ft.

w : Maximum fuselage width, ft.

z_H : Vertical distance from horizontal tail to base of vertical tail, ft.

z_V : Height of vertical tail M.A.C. above body centre line (body axis).

Λ : Sweep back angle of leading edge, deg.

$\Lambda_{c/4}$: Sweep back angle of quarter-chord line, deg.

$\Lambda_{c/2}$: Sweep back angle of mid-chord line, deg.

λ : Taper ratio.

Γ : Geometric dihedral, deg.

C_{L_α}	=	$\partial C_L/\partial\alpha$
$\bar{C}_{D_o}$	=	$C_D - C_L^2/\pi A$
$(\bar{C}_{D_o})_\alpha$	=	$\partial/\partial\alpha\ (C_D - C_L^2/\pi A)$
M	:	Mach number
d	:	Maximum body height at wing-body intersection.
B	=	$\sqrt{1-M^2 \cos^2\Lambda_{c/4}}$
$\mathcal{K}$	:	Ratio of actual wing/tail section lift-curve-slope to 2π
K_H	:	Correction factor for horizontal tail position.
$\bar{X}$	:	Distance from the leading edge of the wing M.A.C. to the airplane C.G.

Subscripts

T	:	Tip
R	:	Root
LE	:	Leading edge
c/4	:	Quarter-chord
c/2	:	Mid-chord
W	:	Wing
WB	:	Wing-plus-body
H	:	Horizontal tail
HB	:	Horizontal tail-plus-body
V	:	Vertical tail

eff	:	effective
av	:	average
(Note	:	Other symbols, when used, have been explained in the text)

SYNOPSIS

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Indian Institute of Technology, Kanpur
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PREDICTION OF DYNAMIC LATERAL STABILITY CHARACTERISTICS

The equations of Lateral-Directional motion of an airplane for the stick-fixed case are developed in terms of the mass, geometric and flight parameters.

The coefficients in these equations are functions of of the lateral stability derivatives. Several methods of estimation of these derivatives and their values estimated by these methods are presented.

Computer programmes have been developed for estimating the time history of important airplane lateral motion parameters.

A sample case, based on the Cessna-182 single-engined airplane, has been worked out in detail, covering the estimation of the stability derivatives and the prediction of the airplane motion in response to a disturbance, by solution of the lateral equations of motion.

1. INTRODUCTION

The calculation of the stick-fixed lateral dynamic characteristics of an airplane requires the solution of the lateral equations of motion, These are a set of three coupled, simultaneous, second order differential equations. The unknown parameters are, the angles of sideslip, roll and yaw and rates of roll and yaw. The coefficients in these equations contain stability derivatives and are functions of the airplane's mass, geometric and flight parameters.

Two methods of solution of the equations of motion are considered. These are, the Laplace Transform method (Appendices A, B and C) and the fourth-order Runge-Kutta method (Appendices D and E).

The Laplace Transform method has the following advantages:-

(a) It provides the roots of the stability quartic.

(b) The period and damping of various modes of lateral motion can be calculated.

(c) It permits calculation of the free motion following any initial conditions, as well as, motion following application of external forces and moments.

(d) It is very useful in further analysis such as frequency response studies by Bode plots etc.

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(d) It is very useful in further analysis such as frequency response studies by Bode plots etc.

Solution obtained by Runge-Kutta method provides only the time history of the unknown parameters. This method was considered for its simplicity and for comparison with the results of the Laplace Transform method.

Several methods of estimation of lateral stability derivatives are available. These are presented and discussed.

It was considered very important to take up a sample case and make a complete analysis. Cessna-182 (Fig.1(a),(b) and (c)) was chosen for this purpose because of the availability of this model in the Department and the possibility of someone else pursuing this programme by obtaining flight data for correlation with the data estimated in the report.

The motion of the airplane as calculated here is in the form of time history of angles of sideslip, roll and yaw and rates of roll and yaw following any initial conditions.

The results can, by proper modification of the programme, be extended to calculate motion resulting from impressed forces and moments which are functions of time.

All the forces and moments in the text are referred to stability system of axes which are defined in Figs.2 and 3.

Typical input/output for the solution of lateral equations of motion and prediction of time history of angles of sideslip, roll and yaw and rates of roll and yaw are

2. EQUATIONS OF LATERAL MOTIONS OF AN AIRPLANE

2.1 The dimensional form of the equations of lateral motion (stick-fixed) are:

$$m\,k_X^2 \frac{d^2\phi}{dt^2} - \frac{\partial L}{\partial p}\frac{d\phi}{dt} + m\,k_{XZ}\frac{d^2\psi}{dt^2} - \frac{\partial L}{\partial r}\frac{d\psi}{dt} - \frac{\partial L}{\partial v}v - L_c = 0 \qquad (2.1)$$

$$m\,k_{XZ}\frac{d^2\phi}{dt^2} - \frac{\partial N}{\partial p}\frac{d\phi}{dt} + m\,k_Z^2\frac{d^2\psi}{dt^2} - \frac{\partial N}{\partial r}\frac{d\psi}{dt} - \frac{\partial N}{\partial v}v - N_c = 0 \qquad (2.2)$$

$$-\frac{\partial Y}{\partial p}\frac{d\phi}{dt} - (\text{Lift})\,\phi + mV\frac{d\psi}{dt} - \frac{\partial Y}{\partial r}\frac{d\psi}{dt} - (\text{Lift})(\tan\gamma)\psi$$

$$+ m\frac{dv}{dt} - \frac{\partial Y}{\partial v}v - Y_c = 0 \qquad (2.3)$$

Dividing equations (2.1) and (2.2) by $\frac{1}{2}\rho V^2 Sb$ and (2.3) by $\frac{1}{2}\rho V^2 S$, the nondimensional form of equations are obtained.

$$2\mu K_X^2\frac{d^2\phi}{ds^2} - \frac{1}{2}C_{l_p}\frac{d\phi}{ds} + 2\mu K_{XZ}\frac{d^2\psi}{ds^2} - \frac{1}{2}C_{l_r}\frac{d\psi}{ds} - C_{l_\beta}\beta - C_{l_c} = 0$$

$$2\mu K_{XZ}\frac{d^2\phi}{ds^2} - \frac{1}{2}C_{n_p}\frac{d\phi}{ds} + 2\mu K_Z^2\frac{d^2\psi}{ds^2} - \frac{1}{2}C_{n_r}\frac{d\psi}{ds} - C_{n_\beta}\beta - C_{n_c} = 0$$

$$-\frac{1}{2}C_{Y_p}\frac{d\phi}{ds} - C_L\phi + 2\mu\frac{d\psi}{ds} - \frac{1}{2}C_{Y_r}\frac{d\psi}{ds} - C_L\tan\gamma\,.\psi + 2\mu\frac{d\beta}{ds}$$

where s = non-dimensional time $(\frac{Vt}{b})$.

Equations (2.4) are multiplied by $(m/\rho Sb)$, the relative density factor μ and are written in the following form

$$\left.\begin{aligned}
&(D^2-l_pD)\ \phi + (K_1D^2-l_rD)\ \psi - l_\beta\beta-l_c = 0\\
&(K_2D^2-n_pD)\ \phi + (D^2-n_rD)\ \psi - n_\beta\beta-n_c = 0\\
&(-y_pD-C_L/2)\ \phi + (D-y_rD-C_L/2(\tan\gamma))\ \psi + (D-y_\beta)\ \beta-y_c = 0
\end{aligned}\right\} \quad (2.5)$$

where $D = d/d\sigma$ and the other coefficients are explained in the list of symbols.

2.2 Solution of Equations of Lateral Motion by Laplace Transform Method:-

Taking Laplace transform of equations (2.5), we have, with the given initial conditions,

$$\left.\begin{aligned}
&(\lambda^3 - l_p\lambda^2)\ \phi_\lambda + (K_1\lambda^3 - l_r\lambda^2)\ \psi_\lambda - l_\beta\lambda\beta_\lambda = r_1\\
&(K_2\lambda^3 - n_p\lambda^2)\ \phi_\lambda + (\lambda^3 - n_r\lambda^2)\ \psi_\lambda - n_\beta\lambda\beta_\lambda = r_2\\
&(-y_p\lambda^2 - \frac{C_L}{2}\lambda)\ \phi_\lambda + \left[\lambda^2 - y_r\lambda^2 - C_L/2(\tan\gamma)\lambda\right]\ \psi_\lambda\\
&\qquad + (\lambda^2 - y_\beta\lambda)\ \beta_\lambda = r_3
\end{aligned}\right\} \quad (2.6)$$

where $r_1 = (\lambda^2 - l_p\lambda)\,\phi_o + (K_1\lambda^2 - l_r\lambda)\,\psi_o$

$$+\lambda(D\phi)_o + K_1\lambda(D\psi)_o + l_c$$

$$r_2 = (K_2\lambda^2 - n_p\lambda)\,\phi_o + (\lambda^2 - n_r\lambda)\,\psi_o$$

$$+ K_2\lambda(D\phi)_o + \lambda(D\psi)_o + n_c$$

$$r_3 = -y_p\lambda\phi_o + (\lambda - y_r\lambda)\,\psi_o + \lambda\beta_o - y_c$$

and $\mathcal{L}(\phi) \doteq \phi_\lambda$ etc.

Solving equations (2.6) ϕ_λ, ψ_λ and β_λ can be expressed as

$$\phi_\lambda = \frac{a_o\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5}{\lambda^2(A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E)} \qquad (2.7)$$

$$\psi_\lambda = \frac{b_o\lambda^5 + b_1\lambda^4 + b_2\lambda^3 + b_3\lambda^2 + b_4\lambda + b_5}{\lambda^2(A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E)} \qquad (2.8)$$

$$\beta_\lambda = \frac{c_o\lambda^4 + c_1\lambda^3 + c_2\lambda^2 + c_3\lambda + c_4}{\lambda(A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E)} \qquad (2.9)$$

The coefficients a_o, .., a_5, b_o,..,b_5, c_o,..c_4 and A, B, C, D and E which are the coefficients of the

characteristic equation,

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0 \tag{2.10}$$

are functions of airplane mass and geometric characteristics and aerodynamic stability derivatives, and the expressions for these coefficients appear on pp. 61-63 (Appendix A).

The expressions for ϕ, ψ and β can now be obtained by taking the inverse Laplace transform of equations (2.7) to (2.9).

Assuming that all roots λ_i of the characteristic equation are distinct, the inverse laplace transform gives

$$\phi = A_1 e^{\sigma' \lambda_1} + A_2 e^{\sigma' \lambda_2} + A_3 e^{\sigma' \lambda_3} + A_4 e^{\sigma' \lambda_4} + A_5 \sigma' + A_6 \tag{2.11}$$

$$\psi = B_1 e^{\sigma' \lambda_1} + B_2 e^{\sigma' \lambda_2} + B_3 e^{\sigma' \lambda_3} + B_4 e^{\sigma' \lambda_4} + B_5 \sigma' + B_6 \tag{2.12}$$

$$\beta = C_1 e^{\sigma' \lambda_1} + C_2 e^{\sigma' \lambda_2} + C_3 e^{\sigma' \lambda_3} + C_4 e^{\sigma' \lambda_4} + C_5 \tag{2.13}$$

and the expressions for rate of roll, p and rate of yaw, r, are obtained by differentiating equations (2.11) and (2.12) respectively.

$$p = \frac{1}{\tau} \left(A_1 \lambda_1 e^{\sigma' \lambda_1} + A_2 \lambda_2 e^{\sigma' \lambda_2} + A_3 \lambda_3 e^{\sigma' \lambda_3} + A_4 \lambda_4 e^{\sigma' \lambda_4} + A_5\right) \tag{2.14}$$

$$r = \frac{1}{\tau}(B_1\lambda_1 e^{\sigma'\lambda_1} + B_2\lambda_2 e^{\sigma'\lambda_2} + B_3\lambda_3 e^{\sigma'\lambda_3} + B_4\lambda_4 e^{\sigma'\lambda_4} + B_5) \tag{2.15}$$

The expressions for the coefficients $A_1, \ldots A_6$, $B_1, \ldots, B_6$, $C_1, \ldots, C_5$ appear on pp. 64 (Appendix A).

3. ESTIMATION OF TIME HISTORY OF LATERAL MOTION

3.1 The process of obtaining the lateral motions of an airplane can be divided into four stages.

<u>Stage I</u>: The values of the following parameters are to be determined/estimated:

(a) Mass characteristics; m, k_{X_0}, k_{Z_0}, η and ρ.

(b) Geometric characteristics; Area S, span b etc.

(c) Flight conditions; velocity V, lift coefficient C_L and flight path angle γ .

(d) Aerodynamic Stability Derivatives.

i) Side-slip derivatives C_{l_β}, C_{n_β}, C_{Y_β}

ii) Roll-rate derivatives C_{l_p}, C_{n_p}, C_{Y_p}

iii) Yaw-rate derivatives C_{l_r}, C_{n_r}, C_{Y_r}

The methods by which these derivatives are estimated, are elaborated in the next section.

(e) In cases where impressed forces are used as disturbances, the values of factors C_{l_c}, C_{n_c}, C_{Y_c} appropriate to the particular problem are to be determined.

<u>Stage II</u>: From the above known quantities, the non-dimensional parameters as used in the calculation of motion are evaluated. From these parameters, the coefficients A, B, C, D and E

Stage III: The roots of the above characteristic equation are then obtained. In the programme used, the method of reduction to cubic has been used.

Stage IV: The appropriate factors $A_1,\ldots A_6$, $B_1,\ldots B_6$, $C_1,\ldots C_5$ are evaluated. Some of these factors may be complex conjugate pairs depending on the nature of the roots λ_i of the characteristic equation. Accordingly the equations of motion are written as follows:

(a) All Roots Real

$$\left.\begin{aligned}
\phi &= \sum_{i=1}^{4} A_i e^{\sigma\lambda_i} + A_5\sigma + A_6 \\
\psi &= \sum_{i=1}^{4} B_i e^{\sigma\lambda_i} + B_5\sigma + B_6 \\
\beta &= \sum_{i=1}^{4} C_i e^{\sigma\lambda_i} + C_5 \\
p &= \frac{1}{\tau}\left[\sum_{i=1}^{4} A_i\lambda_i e^{\sigma\lambda_i} + A_5\right] \\
r &= \frac{1}{\tau}\left[\sum_{i=1}^{4} B_i\lambda_i e^{\sigma\lambda_i} + B_5\right]
\end{aligned}\right\} \qquad (3.1)$$

(b) One Complex Pair of Roots and Two Real Roots

$$\left.\begin{aligned}
\lambda_1 &= R + iI \\
\lambda_2 &= R - iI
\end{aligned}\right\} \quad \text{complex conjugate pair}$$

$$
\left.\begin{aligned}
\phi &= K_A e^{\sigma R} \operatorname{Cos}(\sigma I + w_A) + A_3 e^{\sigma \lambda_3} + A_4 e^{\sigma \lambda_4} + A_5 \sigma + A_6 \\
\psi &= K_B e^{\sigma R} \operatorname{Cos}(\sigma I + w_B) + B_3 e^{\sigma \lambda_3} + B_4 e^{\sigma \lambda_4} + B_5 \sigma + B_6 \\
\beta &= K_C e^{\sigma R} \operatorname{Cos}(\sigma I + w_C) + C_3 e^{\sigma \lambda_3} + C_4 e^{\sigma \lambda_4} + C_5 \\
p &= \frac{1}{\tau}\Big[K_A \sqrt{R^2+I^2}\, e^{\sigma R} \operatorname{Cos}(\sigma I + w_A + \tan^{-1}\tfrac{I}{R}) \\
&\qquad + A_3 \lambda_3 e^{\sigma \lambda_3} + A_4 \lambda_4 e^{\sigma \lambda_4} + A_5 \Big] \\
r &= \frac{1}{\tau}\Big[K_B \sqrt{R^2+I^2}\, e^{\sigma R} \operatorname{Cos}(\sigma I + w_B + \tan^{-1}\tfrac{I}{R}) \\
&\qquad + B_3 \lambda_3 e^{\sigma \lambda_3} + B_4 \lambda_4 e^{\sigma \lambda_4} + B_5 \Big]
\end{aligned}\right\} \qquad (3.2)
$$

where

$$
\left.\begin{aligned}
K_A &= 2\sqrt{R_A^2 + I_A^2} \quad ; \quad w_A = \tan^{-1} \frac{I_A}{R_A} \\
K_B &= 2\sqrt{R_B^2 + I_B^2} \quad ; \quad w_B = \tan^{-1} \frac{I_B}{R_B} \\
K_C &= 2\sqrt{R_C^2 + I_C^2} \quad ; \quad w_C = \tan^{-1} \frac{I_C}{R_C} \\
A_1 &= R_A + I_A i \quad ; \quad A_2 = R_A - I_A i
\end{aligned}\right\} \qquad (3.3)
$$

The steps to determine R_A and I_A have been shown in

(c) Two Pairs of Complex Roots:

$$\lambda_1, \lambda_2 = R \pm iI$$

$$\lambda_3, \lambda_4 = R' \pm iI'$$

$$\left.\begin{aligned}
\phi &= K_A e^{\sigma R} \cos(\sigma I + w_A) + K_A' e^{\sigma R'} \cos(\sigma I' + w_A') + A_5\sigma + A_6 \\
\psi &= K_B e^{\sigma R} \cos(\sigma I + w_B) + K_B' e^{\sigma R'} \cos(\sigma I' + w_B') + B_5\sigma + B_6 \\
\beta &= K_C e^{\sigma R} \cos(\sigma I + w_C) + K_C' e^{\sigma R'} \cos(\sigma I' + w_C') + C_5 \\
p &= \frac{1}{\tau}\Big[K_A\sqrt{R^2+I^2}\, e^{\sigma R} \cos\left(\sigma I + w_A + \tan^{-1}\frac{I}{R}\right) + A_5 \\
&\qquad + K_A'\sqrt{R'^2+I'^2}\, e^{\sigma R'} \cos\left(\sigma I' + w_A' + \tan^{-1}\frac{I'}{R'}\right)\Big] \\
r &= \frac{1}{\tau}\Big[K_B\sqrt{R^2+I^2}\, e^{\sigma R} \cos\left(\sigma I + w_B + \tan^{-1}\frac{I}{R}\right) + B_5 \\
&\qquad + K_B'\sqrt{R'^2+I'^2}\, e^{\sigma R'} \cos\left(\sigma I' + w_B' + \tan^{-1}\frac{I'}{R'}\right)\Big]
\end{aligned}\right\} \quad \ldots (3.4)$$

where quantities with primes are defined in exactly the same way as in (3.3).

The appropriate equations of motion (3.1), (3.2) or (3.4) can now be solved by substituting the values of non-dimensional time factor σ to obtain the time history of lateral motion parameters, ϕ, ψ, β, p or r.

3.2 Motions Resulting from Arbitrary Disturbances:

The motions resulting from arbitrary forcing functions can be obtained from the motions resulting from constant impressed forces and moments by methods explained in Ref.4.

In Ref. 4 it is shown that although component motions of an airplane must be calculated simultaneously, the effects of component disturbances may, by the principle of superposition, be calculated separately and later added in desired proportions. This principle of superposition is elaborated with examples in the same reference.

4. ESTIMATION OF LATERAL STABILITY DERIVATIVES

4.1 GENERAL: Methods of estimation of contribution of all the main components of an airplane to all the derivatives, for airplanes having any sweep angle or aspect ratio, have been presented in various publications.

As long as these methods are based on available flight test or wind tunnel test data on similar designs or theoretical or empirical formulae, the values obtained are suitable only for first approximations of dynamic stability. Extensive wind tunnel test data on scaled or full scale model of the particular configuration will be essential for accurate estimations.

The estimations made here depend on theoretical treatments, based on assumptions of steady roll and yaw or experimental data obtained principally from tests made under conditions of steady roll and yaw.

In the present study, the effect of Mach number on the derivatives is indicated for the subsonic case. But for the sample case of estimation of stability derivatives for the Cessna-182 airplane, the effect of Mach number is neglected. The effects of power are briefly treated in Ref.4, but these effects are also neglected in the present study.

The estimation of stability derivatives has been based on the available formulae and methods of Refs. 1 to 5. The first three are text books, which have drawn material from published and unpublished research papers. The fourth is a summary of the available methods of estimation. The fifth is a very recent publication on the subject and has drawn material generously from 'USAF Stability and Control Datcom' which is one of the most reliable sources in use in U.S.A. for estimation of stability derivatives.

The formulae and methods contained in the above five references have much in common, but in Ref. 5 additional (empirical) correction factors are provided and also geometric twist is accounted for. Wherever there are wide variations among the formulae, it has been so pointed out and the results obtained by these various methods are tabulated in Table 6 (a) and (b) for the two flight conditions considered. The values of derivatives which are provided by the manufactures of Cessna airplane are also given in the same table for comparison. A discussion of these results follows in section 5.

4.2 The Side-slip Derivatives C_{Y_β}, C_{n_β}, C_{l_β} :-

These derivatives may vary widely and often in an unpredictable manner with angles of attack and hence no satisfactory theoretical methods are available. Thus, it is

essential to base these estimates on wind tunnel test data. In the absence of such data, reasonably accurate estimates can be made by applying suitable corrections, to available data for similar designs. Certain theoretical and empirical formulae have been evolved for estimating the contribution of the major airplane components.

The side force derivatives are affected by such components as leading edge high-lift devices, stall control devices, trailing edge flaps, nacelles, external stores like fuel tanks etc., canopies and dorsal and ventral fins. Generally, wind tunnel test data is essential for estimating all these effects. Several research papers dealing with the effects of these individual components are available for example in Ref.4.

4.2.1. <u>Side Force Derivative Due to Side-slip,</u> C_{Y_β}

4.2.1.1 This derivative gives the variation of side force coefficient with a positive side-slip angle β. C_{Y_β} is usually negative. This derivative contributes to the damping of the Dutch Roll mode; thus, large negative values of C_{Y_β} might seem desirable. But large negative values of C_{Y_β}, however, may create a large time lag in the airplane's response and cause it to react sluggishly to the pilot's commands. The main contributions are from the body and the vertical tail. For wings with dihedral, the contribution of the wing may

compared to that of the vertical tail. The body contribution arises mainly because of separation of flow over the fuselage. The main contribution to C_{Y_β} is from the vertical tail. As far as possible an accurate estimate of this contribution is essential, since this factor is used in estimation of vertical tail contribution to several other derivatives. It is specially significant at low angles of attack. It is highly desirable to have tail-off and tail-on wind-tunnel test data for an accurate estimation.

Wing plan form and wing location can cause large differences in side-wash and dynamic pressure at the tail. It is difficult to predict the side-wash effects accurately and hence wind-tunnel test data is very helpful.

4.2.1.2: The contribution of various components are estimated as follows:

$$C_{Y_\beta} = C_{Y_{\beta_W}} + C_{Y_{\beta_B}} + C_{Y_{\beta_V}} \tag{4.1}$$

(a) Wing: Generally this contribution is small. For wings with large dihedral, this contribution is estimated from the empirical formula given in Ref. 5.

$$C_{Y_{\beta_W}} = -0.0001\,|\Gamma|.57.3\ (\text{rad}^{-1}) \tag{4.2}$$

where

Γ = Geometric dihedral, deg.

(b) Body:- A rough estimation of this contribution as given in Ref. 2 is

$$C_{Y_{\beta_B}} = -2\frac{S_b}{S} \tag{4.3}$$

where S_b is an equivalent base area and S is the wing area. S_b lies between zero and $(\pi/4)h^2$ where h is the maximum fuselage depth. But it is very difficult to obtain an exact value of S_b. But since $C_{Y_{\beta_B}}$ is small compared to vertical tail contribution, a high degree of accuracy is not needed. $C_{Y_{\beta_B}}$ is independent of Mach number as per slender body theory.

A better approximation for this contribution is given in Ref. 5 as follows

$$C_{Y_{\beta_B}} = -2K_i\ (S_o/S)\ (\mathrm{rad}^{-1}) \tag{4.4}$$

where

K_i = a wing-body interference factor obtained as a function of the wing location with respect to body i.e., $z_w\ /(d/2)$, from Fig. 4.

where

z_w = the distance from body-centre line to quarter chord

the quarter chord point below the body centre line.

d = maximum body height at wing-body intersection.

S_o = cross-sectional area of the fuselage at the point X_o, along the body where the flow ceases to be potential. The distance X_o is a function of X_1, the body station where dS_X/dX first reaches its maximum negative value. The distances X_o and X_1 are related as in Fig. 6. To find X_1 and X_o, variation of fuselage cross-sectional area along the body centre line has been tabulated in Table 2, and plotted as a function of X (Fig.5). X_1 is determined from this figure and corresponding value of X_o from Fig.6. The value of S_o is then read from Fig.5.

(c) <u>Vertical Tail</u>: A theoretical estimate of this contribution is made from the formula given in Ref.5.

$$C_{Y_{\beta_V}} = -\,kC_{L_{\alpha_V}}\,(1 + d\sigma/d\beta)\,\eta_V\,S_V/S\ (\text{rad}^{-1}) \qquad (4.5)$$

where:

k = an empirical factor depending on the constant $(b_V/2r_1)$ (defined in Fig.7) and λ_V and is obtained from Fig.7.

$C_{L_{\alpha_V}}$ = tail lift-curve-slope based on the effective aspect ratio which is approximately 1.55 times the geometric

aspect ratio (b_V^2/S_V) (Ref.3), and is calculated as follows

$$C_{L_{\alpha_V}} = \frac{2\pi A_{V_{eff}}}{2 + \sqrt{\frac{A_{V_{eff}}^2 \beta^2}{\kappa^2}\left(1 + \frac{\tan^2 \Lambda_{V_{c/2}}}{\beta^2}\right) + 4}} \quad (4.6)$$

where:

$\beta = \sqrt{1 - M^2}$

κ = ratio of actual average tail section lift-curve-slope to 2π

$$A_{V_{eff}} = \left(\frac{A_{V(B)}}{A_V}\right) A_V \left\{1 + K_H \left(\frac{A_{V(HB)}}{A_{V(B)}} - 1\right)\right\} \quad (4.7)$$

$\left(\frac{A_{V(B)}}{A_V}\right)$ = ratio of the aspect ratio of the vertical panel in the presence of the body to that of the isolated panel and is obtained from Fig. 8 as a function of $b_V/2r_1$ and λ_V.

$\frac{A_{V(HB)}}{A_{V(B)}}$ = ratio of the aspect ratio of the vertical panel in the presence of the horizontal tail and body to that of the panel in the presence of the body alone. This ratio is obtained from Fig.9 as a function of X/C_V and Z_H/b_V (X, C and Z

defined in the same figure)

K_H = a factor accounting for the relative size of the horizontal and vertical tails and is obtained from Fig. 10 as a function of S_H/S_V.

The effect of sidewash and variation of dynamic pressure at the vertical tail is calculated from the empirical formula

$$(1 + d\sigma/d\beta)\eta_V = 0.724 + 3.06 \frac{S_V/S}{1+\cos\Lambda_{c/4}} + 0.4 \frac{z_w}{d} + .009A \tag{4.8}$$

4.2.1.3 References 1 to 3 suggest utilisation of wind tunnel test data for estimation of C_{Y_β}. According to Ref.4 interference effects are so large that a generalized formula would not be completely satisfactory. A method of applying correction to the data on a similar design has, instead, been recommended. The method given above from Ref.5 probably gives the most accurate values since interference effects based on experimental results are included.

4.2.2 Variation of Rolling Moment Coefficient with Sideslip Angle, C_{l_β}

4.2.2.1: This derivative is also called the 'dihedral effect'. C_{l_β} may be positive or negative and for some

configurations it has large variations. It is a very important design parameter. It aids in damping both the Dutch Roll mode and the spiral mode. Small negative values of C_{l_β} give favourable Dutch Roll damping characteristics, but for spiral stability large negative values are necessary.

The main contributions to C_{l_β} are from the wing-body, horizontal tail and the vertical tail.

(a) Wing-Body Contribution: This contribution is again due to three aerodynamic effects.

i) Wing geometric dihedral

ii) Wing sweep angle

iii) Wing position on the fuselage.

(i) When an airplane with positive dihedral side-slips, the right wing will be at a higher angle of attack ($\Delta\alpha = \beta\Gamma$) than the left wing. The increased lift on the right wing and the corresponding decreased lift on the left wing, cause a negative rolling moment. This rolling moment is proportional to the geometric dihedral.

(ii) Wing sweep angle has the same effect as geometric dihedral. This effect is proportional to the lift coefficient. For large C_L i.e., at low speeds, C_{l_β} due to sweep can be very large negatively and becomes small negatively at high speeds. This poses a serious design problem for high speed airplanes with swept back wings.

iii) The fuselage contribution to C_{l_β} is also very significant. The flow field is considerably modified by the shape of the body. The cross flow component of the stream induces vertical velocities which, combined with the main stream velocity alters the local angle of attack of the wing. Consequently, a high wing configuration produces a negative rolling moment and vice versa. The effect also depends on the fuselage length ahead of the wing, its cross-sectional shape and planform and location of the wing.

(b) Horizontal Tail Contribution. The horizontal tail contribution can be explained in exactly the same manner as for the wing. Except when the horizontal tail size is considerable, this contribution is generally negligible.

(c) Vertical Tail Contribution. The sideslipping airplane produces side forces which give rise to significant rolling moment if the vertical tail aerodynamic centre is offset considerably from the rolling axis (i.e., if z is large).

4.2.2.2 : The estimation of the contribution to C_{l_β} of various airplane components is carried out as follows:

$$C_{l_\beta} = C_{l_{\beta_{WB}}} + C_{l_{\beta_H}} + C_{l_{\beta_V}} \qquad (4.9)$$

(a) Wing-Body: The wing-body contribution is calculated by the following formula (Ref.5).

$$C_{l_{\beta_{WB}}} = 57.3 \left[C_{L_{WB}} \left\{ \left(\frac{C_{l_\beta}}{C_L}\right)_{\Lambda_{c/2}} K_{M_\Lambda} K_f + \left(\frac{C_{l_\beta}}{C_L}\right)_A \right\} + \Gamma \left\{ \left(\frac{C_{l_\beta}}{\Gamma}\right) K_{M_\Gamma} + \frac{\Delta C_{l_\beta}}{\Gamma} \right\} + (\Delta C_{l_\beta})_{z_w} + \theta \tan \Lambda_{c/4} \frac{\Delta C_{l_\beta}}{\theta \tan \Lambda_{c/4}} \right] \text{rad}^{-1} \qquad (4.10)$$

where:

$C_{L_{WB}} \simeq C_L$ = airplane steady state lift coefficient

$\left(\frac{C_{l_\beta}}{C_L}\right)_{\Lambda_{c/2}}$ = the effect of wing sweep contribution and is obtained from Fig. 11.

K_{M_Λ} = compressibility correction to sweep and is obtained from Fig. 12.

K_f = a fuselage correction factor obtained from Fig. 13.

$\left(\frac{C_{l_\beta}}{C_L}\right)_A$ = contribution due to aspect ratio and is obtained from Fig. 14.

$\left(\frac{C_{l_\beta}}{\Gamma}\right)$ = wing dihedral effect obtained from Fig. 15.

K_{M_Γ} = compressibility correction factor to dihedral and is obtained from Fig. 16.

$\frac{\Delta C_{l_\beta}}{\Gamma}$ = body induced effect on the wing height as a function of the cross-section of the body and is given by the empirical formula.

$$\frac{\Delta C_{l_\beta}}{\Gamma} = -0.0005\ A\ (\frac{d_{av}}{b})\ \text{deg.}^{-2} \qquad (4.11)$$

where

$$d_{av} = \sqrt{\frac{4(\text{average cross-sectional area})}{\pi}} \qquad (4.12)$$

The average cross-sectional area is calculated from Table 2 as a weighted average.

$(\Delta C_{l_\beta})_{z_w}$ = another body induced effect on the wing height as a function of z_w and d_{av} i.e., the location of the wing w.r.t. fuselage and is given by

$$(\Delta C_{l_\beta})_{z_w} = \frac{1.2\sqrt{A}}{57.3}\ (\frac{z_w}{b})\ (\frac{2d_{av}}{b})\ (\text{deg}^{-1}) \qquad (4.13)$$

$(\frac{\Delta C_{l_\beta}}{\theta\ \tan\Lambda_{c/4}})$ = a wing twist correction factor obtained from Fig. 17.

Refs. 1, 2 and 4 give a formula for the wing-body contribution which is almost similar to the above.

$$C_{l_{\beta_{WB}}} = \left(\frac{C_{l_\beta}}{C_L}\right) C_L + C_{l_{\beta_\Gamma}} \cdot \Gamma + 1.2\sqrt{A}\ \frac{Z_W}{b}\ \frac{(h_{av} + w_{av})}{b} \tag{4.14}$$

where the values of $\left(\frac{C_{l_\beta}}{C_L}\right)$, $C_{l_{\beta_\Gamma}}$ are given in graphs and

h_{av} = average fuselage height at wing root $\simeq h$

w_{av} = average fuselage width at wing root. $\simeq w$

The formula of Ref. 5 takes into account the effect of sweep and geometric twist.

Ref. 3 gives the following formula for the wing-body contribution

$$C_{l_{\beta_{WB}}} = (C_{l_\beta})_W + (\Delta C_{l_\beta})_1 + (\Delta C_{l_\beta})_2 + (C_{l_\beta})_{W,\Gamma=0} \tag{4.15}$$

The wing dihedral contribution $(C_{l_\beta})_W$ is given by

$$(C_{l_\beta})_W = \left(\frac{C_{l_\beta}}{\Gamma}\right)\Gamma + (\Delta C_{l_\beta})_{\text{tip shape}} \tag{4.16}$$

$(\Delta C_{l_\beta})_1$ is the contribution to C_{l_β} due to wing-fuselage interference and is given below.

	(ΔC_{l_β})
High wing	$-.0006$
Mid wing	0
Low wing	$+.0008$

$(\Delta C_{l_\beta})_2$ is the contribution to C_{l_β} due to wing-vertical tail interference and is given below

	$(\Delta C_{l_\beta})_2$
High wing	.00016
Mid wing	0
Low wing	-.00016

$$(C_{l_\beta})_{W,\Gamma=0} = C_L\left[-\frac{k(.71\lambda + .29)}{A\lambda} + .05\right] - (\text{rad}^{-1}) \tag{4.17}$$

where

$k = 1.0$ for straight wing tips

$k = 1.5$ for round wing tips.

The above formula is a revised one for unswept, elliptical wings with zero dihedral taking into consideration changes in taper ratio, for the wing contribution to C_{l_β}.

(b) Horizontal Tail: The contribution of the horizontal tail is calculated from

$$C_{l_{\beta_H}} = C_{l_{\beta_{HB}}}\left(\frac{\eta S_H\, b_H}{S\, b}\right) \text{rad}^{-1} \tag{4.18}$$

where $C_{l_{\beta_{HB}}}$ is calculated by treating the fuselage-horizontal tail in the same manner as fuselage-wing has been treated. Significance of $C_{l_{\beta_H}}$ depends on the magnitude of $(S_H\, b_H/Sb)$.

(c) Vertical Tail: The vertical tail contribution is estimated from the following formula (Ref.5)

$$C_{l_{\beta_V}} = C_{Y_{\beta_V}} \cdot \frac{z}{b}$$

$$= C_{Y_{\beta_V}} \cdot \frac{(z_V \cos\alpha - l_V \sin\alpha)}{b} \qquad (4.19)$$

where z_V and l_V are as defined in Fig. 18.

Refs. 1 and 3 give the following slightly different formula for $C_{l_{\beta_V}}$

$$C_{l_{\beta_V}} = - C_{L_{\alpha_V}} \frac{S_V\, z}{S\, b} \cdot \eta_V \qquad (4.20)$$

4.2.3 Variation of Yawing Moment Coefficient with Sideslip Angle, C_{n_β}.

4.2.3.1 This is also called the 'static directional' or 'weather cock' derivative. It is a measure of the tendency of the airplane to align itself in sideslip, like a weather cock, with the relative wind. It is one of the most important parameters in design. The value of C_{n_β} determines primarily the Dutch Roll natural frequency and affects the spiral stability of the airplane. It varies considerably with tail size. For stability and good flying qualities

C_{n_β} should be as high as practically possible. The vertical tail, fuselage and wing contribute to C_{n_β}, but the tail contribution dominates. For positive sideslip, the vertical tail causes a positive yawing moment and the fuselage contribution is generally a negative yawing moment. The wing contribution is usually positive but generally neglected. The value of C_{n_β} must be positive for stability.

4.2.3.2 The contribution of the various components to C_{n_β} is estimated as follows

$$C_{n_\beta} = C_{n_{\beta_W}} + C_{n_{\beta_B}} + C_{n_{\beta_V}} \tag{4.21}$$

(a) Wing: Wing contribution is generally very small and neglected. However, Refs. 1 and 4 give the following formula for $C_{n_{\beta_W}}$

$$C_{n_{\beta_W}} = C_L^2 \left(\frac{C_{n_\beta}}{C_L^2}\right) \tag{4.22}$$

whereas, Ref. 2 gives an additional term $C_L \Gamma \left(\frac{C_{n_\beta}}{C_L \Gamma}\right)$, where, $\left(\frac{C_{n_\beta}}{C_L \Gamma}\right) \simeq 0.075$ and $\left(\frac{C_{n_\beta}}{C_L^2}\right)$ can be obtained from Ref. 4.

Ref. 3. gives the following formula

$$C_{n_{\beta_W}} = .00006\ (\Lambda)^{1/2}\ \mathrm{deg}^{-1} \qquad (4.23)$$

$$(\Lambda \text{ in deg})$$

which for unswept wings is zero.

(b) Body: The fuselage contribution based on slender body theory gives (Ref.2)

$$C_{n_{\beta_{Fus}}} = -2\,\frac{Vol_f}{Sb} \qquad (4.24)$$

where Vol_f is the volume of an equivalent circular body having the same side-view as the real fuselage, i.e.,

$$Vol_f = \frac{\pi}{4}\int d_x^2\,dx \simeq S_{B_S} \times l_B \qquad (4.25)$$

But the slender body theory tends to over-estimate the fuselage instability, since it neglects the separation at the rear of the body. The body contribution including the interference effect of the wing on the body is estimated from the formula (Ref.5)

$$C_{n_{\beta_B}} = -57.3\,K_N\,K_{Rl}\,\frac{S_{B_S}}{S}\cdot\frac{l_B}{b}\ (\mathrm{rad}^{-1}) \qquad (4.26)$$

where

K_N = an empirical factor for body and body and wing interference effects and is obtained from

Fig. 19. The factor K_N depends on the shape of the body, position of the C.G., the cross-section of the body etc. All these factors for Cessna are shown in Table 3.

K_{Rl} = a Reynold's number factor for the fuselage and is obtained from Fig. 20.

S_{B_S} = body side area

l_B = length of the body

The body contribution given above is valid for all subsonic Mach numbers.

Refs. 1, 2, and 4 give the following formula for the body contribution

$$C_{n_{\beta_B}} = -1.3 \frac{(\text{Fuselage volume})}{Sb} \cdot \left(\frac{h_{av}}{w_{av}}\right) \quad (4.27)$$

The fuselage volume can be found by

$\int_0^{l_B} S_X X\, dX$, or can be approximated as $(l_B \times \pi \frac{d_{av}^2}{4})$.

Ref. 3 gives the following formula for the body contribution.

$$C_{n_{\beta_B}} = \frac{-0.96\, K_\beta}{57.3} \left(\frac{S_{B_S}}{S}\right) \left(\frac{l_B}{b}\right) \left(\frac{h_1}{h_2}\right)^{1/2} \left(\frac{w_2}{w_1}\right)^{1/3} \quad (4.28)$$

where

K_β = an empirical constant obtained from Fig.21 as a function of l_B/h and X_{CG}/l_B.

X_{CG} = the distance from nose to C.G.

h_1, h_2, w_1, w_2 are defined in Fig. 21.

(c) Vertical Tail : The contribution of vertical tail to C_{n_β} is estimated as follows (Ref.5)

$$C_{n_{\beta_V}} = - C_{Y_{\beta_V}} \cdot \frac{\bar{l}}{b}$$

$$= - C_{Y_{\beta_V}} \frac{(l_V \cos \alpha + z_V \sin \alpha)}{b} \qquad (4.29)$$

4.3 Roll Rate Derivatives:

When an airplane rolls with angular velocity p, about its x-axis, the motion of its wing tip is helical with a non-dimensional helix angle equal to $\frac{pb}{2V}$. This motion affects the airflow along the wing and tail surfaces as illustrated in Fig.22, for two points, viz., wing tip and fin tip. These angle-of-attack changes introduce perturbation in the aerodynamic forces and moments. Further, the changes in the wing load distribution causes a modification of trailing vortex sheet. The vorticity distribution is no longer symmetrical and a sidewash is

induced on the vertical tail. Further, the trailing vortex sheet is no longer flat but helical. These sidewash effects have been studied theoretically and experimentally.

4.3.1 Variation of Side Force Coefficient with Roll Rate, C_{Y_p}:

4.3.1.1 This is not an important derivative in estimating the dynamic stability characteristics of an airplane. The main contribution is from the vertical tail, even though for some configurations the wing contribution may be significant. The sign of C_{Y_p} may be either positive or negative.

4.3.1.2 The main contribution to C_{Y_p} is from the vertical tail and is estimated from the relation

$$C_{Y_p} \simeq C_{Y_{p_V}} = 2 \frac{z}{b} C_{Y_{\beta_V}}$$

$$= 2 \frac{(z_V \operatorname{Cos} \alpha - l_V \operatorname{Sin} \alpha)}{b} C_{Y_{\beta_V}} \qquad (4.30)$$

In the above relation, the effect of sidewash is neglected. Ref. 4 gives the following modified relation which approximately accounts for the effects of wing side- wash

$$C_{Y_p} = 2\left[\frac{z_V \operatorname{Cos}\alpha - l_V \operatorname{Sin}\alpha - z_V}{b}\right] C_{Y_{\beta_V}} \qquad (4.31)$$

If the effects of sidewash can be estimated the following relation from Ref. 1 can be used.

$$C_{Y_{p_V}} = C_{L_{\alpha_V}} \frac{S_V}{S}\left[-\frac{2}{b}(z_V \operatorname{Cos}\alpha - l_V \operatorname{Sin}\alpha) + \frac{\partial\sigma'}{\partial(\frac{pb}{2V})}\right] \qquad (4.32)$$

An estimation of the effect of sidewash can be made from the graph in Ref.6.

Refs. 2 and 3 neglect the value of C_{Y_p}.

4.3.2 <u>Variation of Rolling Moment Coefficient with Roll Rate</u>, C_{l_p}

4.3.2.1 This derivative is also known as the 'damping-in-roll' derivative, since, it represents the resistance of the airplane to rolling. When an airplane rolls, the angle of attack on the down-going wing is increased and that on the up-going wing is decreased. This changes the lift distribution on the two wings and a rolling moment, which opposes the initial disturbance, is produced.

C_{l_p} is a negative constant so long as the local angle of attack remains below the stalling angle. If it exceeds, C_{l_p} may become zero or even positive and the wing may 'auto rotate'.

The main contribution to C_{l_p} is from the wings. In case the horizontal tail is very large, its contribution may not be negligible. The contribution of the vertical tail is usually small. C_{l_p} is the principal determinant of the damping-in-roll characteristics of the aircraft.

4.3.2.2 The contribution to C_{l_p} of the various components is estimated as follows

$$C_{l_p} = C_{l_{p_{WB}}} + C_{l_{p_H}} + C_{l_{p_V}} \qquad (4.33)$$

(a) Wing-Body: The wing-body contribution is estimated from the following relation (Ref.5)

$$C_{l_{p_{WB}}} \simeq C_{l_{p_W}} = \left(\frac{\beta C_{l_p}}{\kappa}\right) \frac{\kappa}{\beta} \qquad (4.34)$$

where

$\left(\frac{\beta C_{l_p}}{\kappa}\right)$ = roll damping parameter obtained from Fig.23 as a function of $\Lambda_\beta = \tan^{-1}(\tan \Lambda_{c/4}/\beta)$ (deg.)

κ = ratio of average wing section lift curve slope, $C_{l_{\alpha_W}}$, to 2π.

Based on strip-theory technique, Ref.2 gives the following relation for the wing contribution

$$C_{l_{p_W}} = \frac{C_{L\alpha}}{12}\left(\frac{1+3\lambda}{1+\lambda}\right) \qquad (4.35)$$

This generally provides a higher negative value for C_{l_p}.

Another relation given in Ref.4 is as follows

$$C_{l_{p_W}} = (C_{l_p})_{C_L=0}\,\frac{(C_{L_\alpha})_{C_L}}{(C_{L_\alpha})_{C_L=0}} - \frac{1}{8}\,C_L^2\left\{1+2\,\sin^2\Lambda\,\frac{(A+2\,\cos\Lambda)}{(A+4\,\cos\Lambda)}\right\}$$

$$- \frac{1}{8}\left(C_D - \frac{C_L^2}{\pi A}\right) \qquad (4.36)$$

where

$$(C_{l_p})_{C_L=0} = (C_{l_p})_{C_L=0;\,a=2\pi}\left[\frac{A+4\,\cos\Lambda}{\frac{A}{K}+4\,\cos\Lambda}\right] \qquad (4.37)$$

and $(C_{l_p})_{C_L=0;\;a=2\pi}$ is obtained from charts of Ref.4 as a function of aspect ratio, sweep angle and taper ratio.

Ref.3 reproduces a chart from NACA-TR 635, which gives the values of $C_{l_{p_W}}$ as a function of aspect ratio and taper ratio.

(b) <u>Horizontal Tail</u>: The contribution to C_{l_p} of an extremely large horizontal tail can be significant. It is calculated

from the relation (Ref.5)

$$C_{l_{p_H}} = 0.5 \, (C_{l_p})_H \frac{S_H}{S} \left(\frac{b_H}{b}\right)^2 \qquad (4.38)$$

where $(C_{l_p})_H$ is the value of C_{l_p} based on the horizontal tail geometry and is found from equation 4.34. The factor 0.5 is introduced to account for the rotation of flow produced by the wing (Ref.4).

(c) Vertical Tail: The contribution of the vertical tail is estimated from the relation (Ref.5)

$$C_{l_{p_V}} = 2 \left(\frac{z_V}{b}\right)^2 C_{Y_{\beta_V}} \qquad (4.39)$$

The above formula is modified (Ref.4) to allow for approximate correction for the effect of the wing on the damping-in-roll of conventionally located vertical tail surface

$$C_{l_{p_V}} = 2 \left(\frac{z}{b}\right) \left(\frac{z}{b} - \frac{z_V}{b}\right) C_{Y_{\beta_V}}$$

$$\cong \frac{2(z_V \operatorname{Cos} \alpha - l_V \operatorname{Sin} \alpha)}{b^2} (z_V \operatorname{Cos}\alpha - l_V \operatorname{Sin}\alpha - z_V) \times C_{Y_{B_V}} \qquad (4.40)$$

4.3.3 Variation of Yawing Moment Coefficient with Roll Rate, C_{n_p}:

4.3.3.1 This is one of the cross derivatives that cause coupling between the rolling and yawing motions of an airplane. The main contributions are from wing and vertical tail. The wing contribution is a negative yawing moment. The vertical tail contribution may be positive or negative depending on the tail geometry, the angle of attack and sidewash from the wing. C_{n_p} affects the Dutch Roll damping in that, the larger its negative value, the less Dutch Roll damping. Thus, a positive C_{n_p} is desirable.

The wing contribution consists of two parts. The first part is due to change in profile drag due to change in wing angle of attack. For a positive roll, the angle of attack increases on the right wing and decreases on the left wing, which normally causes an increase in the profile drag on the former and a decrease on the latter, thus producing a yawing moment.

The second part is due to the fore and aft inclination of the lift vector caused by rolling. The lift vector on the right wing is inclined forward and that on the other, backward. This causes a negative yawing moment proportional to C_L.

4.3.3.2. The contribution of wing and vertical tail to C_{n_p} is estimated as follows.

$$C_{n_p} = C_{n_{p_W}} + C_{n_{p_V}} \tag{4.41}$$

(a) Wing: The wing contribution is in two parts

$$C_{n_{p_W}} = \frac{(\Delta C_{n_p})_1}{(C_L)} C_L + \frac{(\Delta C_{n_p})_2}{(C_{D_o})_\alpha} \cdot (C_{D_o})_\alpha \tag{4.42}$$

$$= (C_{n_p})_1 + (C_{n_p})_2$$

Strip theory using two dimensional flow assumptions lead to the formula (Ref.2)

$$\frac{(\Delta C_{n_p})_1}{(C_L)} = -\frac{1}{12}\left(\frac{1+3\lambda}{1+\lambda}\right) \tag{4.43}$$

$$\frac{(\Delta C_{n_p})_2}{(C_{D_o})_\alpha} = \frac{1}{12}\frac{(1+3\lambda)}{(1+\lambda)} \tag{4.44}$$

These formulas are good only for a first approximation and give an over estimation of $(C_{n_p})_1$ even for unswept wings. For low aspect ratio and high sweep angles, the effect is more pronounced. The estimate for $(C_{n_p})_2$ is satisfactory except for low aspect ratio wings.

Ref. 4 provides empirical charts which give a more accurate estimation $(\Delta C_{n_p})_1$ and $(\Delta C_{n_p})_2$

from Fig. 24.

Ref. 5 provides the following relation for the wing contribution. This formula takes into account the wing sweep, and geometric twist.

$$C_{n_{p_W}} = - C_{l_{p_W}} \tan\alpha - \left[- C_{l_p} \tan\alpha - \left(\frac{C_{n_p}}{C_L}\right)_{\substack{C_L=0\\M}} \cdot C_L \right]$$

$$+\left(\frac{\Delta C_{n_p}}{\theta}\right)\theta + \left(\frac{\Delta C_{n_p}}{\alpha_{\delta_f}\ \delta_f}\right) \alpha_{\delta_f}\ \delta_f \qquad (4.45)$$

where:

$C_{l_{p_W}}$ = wing contribution to C_{l_p} calculated from (4.34)

α = wing angle of attack

C_L = wing lift coefficient

$\cong$ total airplane lift coefficient.

$\left(\frac{C_{n_p}}{C_L}\right)_{\substack{C_L=0\\M}}$ = variation of the yawing moment coefficient due to roll rate at zero lift, given by

$$\left(\frac{C_{n_p}}{C_L}\right)_{\substack{C_L=0\\M}} = \frac{A + 4\cos\Lambda_{c/4}}{AB + 4\cos\Lambda_{c/4}} \left[\frac{AB+\frac{1}{2}(AB+\cos\Lambda_{c/4})\tan^2\Lambda_{c/4}}{A+\frac{1}{2}(A+\cos\Lambda_{c/4})\tan^2\Lambda_{c/4}}\right] \cdot$$

$$\left(\frac{C_{n_p}}{C_L}\right)_{\substack{C_L=0\\M=0}} \qquad (4.46)$$

where $B = \sqrt{1 - M^2 \cos^2 \Lambda_{c/4}}$

and $\left(\frac{C_{n_p}}{C_L}\right)_{\substack{C_L=0\\M=0}}$ = variation of the low speed yawing moment coefficient due to roll rate at zero lift, given by

$$\left(\frac{C_{n_p}}{C_L}\right)_{\substack{C_L=0\\M=0}} = -\frac{1}{6}\,\frac{A+6(A+\cos\Lambda_{c/4})\left(\frac{\bar{X}}{\bar{c}}\frac{\tan\Lambda_{c/4}}{A}+\frac{\tan^2\Lambda_{c/4}}{12}\right)}{A+4\cos\Lambda_{c/4}}\ (\text{rad}^{-1}) \qquad (4.47)$$

where:

$\bar{X}$ = distance from the C.G. to the aerodynamic centre, positive when the a.c. is aft of the C.G. and $\bar{C}$ is the m.A.C.

$\left(\frac{\Delta C_{n_p}}{\theta}\right)$ = effect of linear wing twist and is obtained from Fig. 25.

θ = geometric twist in deg.

$\left(\frac{\Delta C_{n_p}}{\alpha_{\delta_f}\ \delta_f}\right)$ = effect of symmetric flap deflection (Ref.5)

δ_f = streamwise flap deflection in deg.

α_{δ_f} = two dimensional lift effectiveness parameter (Ref.5).

The two methods mentioned above for the wing contribution could yield different values and the selection of the method to be used should probably depend on the type of airplane for which one is more applicable than the other.

Using an elliptical lift distribution, Ref.3 gives the following formula

$$(C_{n_p})_W = -\frac{C_L}{8} \qquad (4.48)$$

(b) Vertical Tail: The vertical tail contribution is estimated from the following relation (Ref.5)

$$C_{n_{p_V}} = -2\,(1/b)\,\left(\frac{z-z_V}{b}\right)\,C_{Y_{\beta_V}}$$

$$= -2\,\frac{(l_V \cos\alpha + z_V \sin\alpha)}{b}\,\frac{(z_V \cos\alpha - l_V \sin\alpha - z_V)}{b} \times C_{Y_{\beta_V}} \qquad (4.49)$$

Ref. 1 gives an almost similar formula for the tail contribution

$$C_{n_{p_V}} = C_{L_{\alpha_V}}\,\frac{S_V}{S}\cdot\frac{1}{b}\cdot\left(2.\frac{z}{b} - \frac{\partial\sigma}{\partial(\frac{pb}{2V})}\right) \qquad (4.50)$$

4.4 Yaw Rate Derivatives

When a yaw rate, r, is super imposed on the airplane motion, the velocity field is significantly changed. For a

positive yaw rate, the velocity vector on the left wing increases and that on the right wing correspondingly decreases. This, in turn, increases the aerodynamic forces on the left wing and decrease them on the right wing. This lift distribution leads to an unsymmetrical trailing vortex sheet and sidewash at the tail.

4.4.1 Variation of Side Force Coefficient with Yaw Rate, C_{Y_r} :

4.4.1.1 This derivative which normally is of minor importance can be calculated from the relation (Ref.5)

$$C_{Y_r} \simeq C_{Y_{r_V}} = - 2 \frac{l}{b} C_{Y_{\beta_V}}$$

$$= - 2 \frac{(l_V \text{ Cos } \alpha + z_V \text{ Sin } \alpha)}{b} C_{Y_{\beta_V}} \qquad (4.51)$$

since the only significant contribution is from the vertical tail.

4.4.2 Variation of Rolling Moment Coefficient with Yaw Rate, C_{l_r}

4.4.2.1 This is another cross derivative. For a positive yaw rate, the asymmetric load distribution also produces a positive

rolling moment, which is proportional to original C_L. Aspect ratio, sweep, taper ratio, dihedral and twist are all important factors. The use of partial span high lift devices also affects this derivative. The main contributions are from the wing and the vertical tail. If the horizontal tail is very large, its contribution is significant and can be calculated in the same way as wing contribution and applying necessary corrections for effective sideslip. The vertical tail causes rolling moment proportional to its height above the roll axis.

C_{l_r} has little effect on the Dutch Roll damping but is quite important to the spiral mode. For spiral stability, it is desirable that C_{l_r} be as small a positive number as possible.

4.4.2.2 Ref. 3 gives a very simple approximation for C_{l_r} as

$$C_{l_r} = \frac{C_L}{4} \tag{4.52}$$

For a better estimate, the contributions of various components are calculated as follows

$$C_{l_r} = C_{l_{r_W}} + C_{l_{r_V}} \tag{4.53}$$

(a) <u>Wing</u>: The wing contribution is estimated from the following relation (Ref.5)

$$C_{l_{r_W}} = C_L \left(\frac{C_{l_r}}{C_L}\right)_{\substack{C_L=0 \\ M}} + \left(\frac{\Delta C_{l_r}}{\Gamma}\right) \frac{\Gamma}{57.3} + \left(\frac{\Delta C_{l_r}}{\theta}\right) \theta$$

$$+ \left(\frac{\Delta C_{l_r}}{\alpha_{\delta_f} \delta_f}\right) \alpha_{\delta_f} \delta_f \ (\text{rad}^{-1}) \qquad (4.54)$$

where:

$\left(\frac{C_{l_r}}{C_L}\right)_{\substack{C_L=0 \\ M}}$ = variation of rolling moment coefficient due to yaw rate at zero lift, given by

$$\left(\frac{C_{l_r}}{C_L}\right)_{\substack{C_L=0 \\ M}} = \left\{ \frac{1 + \frac{A(1-B^2)}{2B(AB+2\cos\Lambda_{c/4})} + \frac{AB+2\cos\Lambda_{c/4}}{AB+4\cos\Lambda_{c/4}} \cdot \frac{\tan^2\Lambda_{c/4}}{8}}{1 + \frac{A+2\cos\Lambda_{c/4}}{A+4\cos\Lambda_{c/4}} \tan^2\Lambda_{c/4}} \right\}$$

$$\times \left(\frac{C_{l_r}}{C_L}\right)_{\substack{C_L=0 \\ M=0}} \qquad (4.55)$$

and $\left(\frac{C_{l_r}}{C_L}\right)_{\substack{C_L=0 \\ M=0}}$ = Variation of low speed rolling moment coefficient due to yaw rate at zero lift and is obtained from Fig.26 as a function of aspect ratio, sweep of the quarter chord and taper ratio.

$\frac{\triangle C_{l_r}}{\Gamma}$ = increment in C_{l_r} due to dihedral, given by

$$\frac{\triangle C_{l_r}}{\Gamma} = \frac{1}{12} \frac{\pi A \sin \Lambda_{c/4}}{A + 4 \cos \Lambda_{c/4}} \tag{4.56}$$

$\left(\frac{\triangle C_{l_r}}{\theta}\right)$ = increment in C_{l_r} due to wing twist and is obtained from Fig.27

$\left(\frac{\triangle C_{l_r}}{\alpha_{\delta_f} \delta_f}\right)$ = effect of symmetric flap deflection (Ref.5)

and α_{δ_f} and δ_f are defined earlier.

Refs.1a and 4 give the value of $\left(\frac{C_{l_r}}{C_L}\right)$ as a function of aspect ratio, sweep and taper ratio in the form of charts.

(b) Vertical Tail: The vertical tail contribution can be estimated from the relation (Ref.5)

$$C_{l_{r_V}} = -2 \left(\frac{l}{b}\right) \left(\frac{z}{b}\right) . C_{Y_{\beta_V}}$$

$$= -2 \frac{(l_V \cos \alpha + z_V \sin \alpha)}{b} \frac{(z_V \cos \alpha - l_V \sin \alpha)}{b} \times C_{Y_{\beta_V}} \tag{4.57}$$

(c) Horizontal Tail: Ref.2 gives a relation for estimating the horizontal tail contribution

$$C_{l_{r_H}} = \left[(C_{l_r})_H - 2\frac{l}{b} C_{l_{\beta_H}} \right] \frac{b_H S_H}{b S} \qquad (4.58)$$

where $(C_{l_r})_H$ is calculated as for the wing from (4.54) and (4.55).

4.4.3 Variation of Yawing Moment Coefficient with Yaw Rate, C_{n_r}:

4.4.3.1 This is also called the 'damping-in-yaw' derivative, since it represents the resistance of the airplane to yawing. The unsymmetric aerodynamic forces cause an increase in the profile and induced drag on the left wing with a corresponding decrease on the right wing. This produces a negative yawing moment. The effect depends on aspect ratio, taper ratio and sweep back. The side force on the vertical tail also produces a negative yawing moment. The main contributions are from the wing and vertical tail. C_{n_r} is the main contributor to the damping of the Dutch Roll mode and also plays a significant role in determining spiral stability. It is a very important parameter in airplane lateral stability characteristics. For best effects in each of these modes, large negative values of C_{n_r} are desirable.

4.4.3.2 The contributions of wing and tail are estimated as follows

$$C_{n_r} = C_{n_{r_W}} + C_{n_{r_V}} \tag{4.59}$$

(a) Wing: The wing contribution is estimated from the relation

$$C_{n_{r_W}} = \left(\frac{C_{n_r}}{C_L^2}\right) C_L^2 + \left(\frac{C_{n_r}}{\bar{C}_{D_o}}\right) \bar{C}_{D_o} \tag{4.60}$$

where:

$\left(\frac{C_{n_r}}{C_L^2}\right)$ = a factor accounting for the variation of induced drag and is obtained from Fig.28.

$\left(\frac{C_{n_r}}{\bar{C}_{D_o}}\right)$ = a factor accounting for the variation of profile drag and is obtained from Fig. 29.

$\bar{C}_{D_o}$ = wing profile drag coefficient given by

$$\bar{C}_{D_o} = \left(C_D - \frac{C_L^2}{\pi A}\right) \tag{4.61}$$

(b) Vertical Tail: The vertical tail contribution is estimated from the relation (Ref.5)

$$C_{n_{r_V}} = 2\left(\frac{l}{b}\right)^2 C_{Y_{\beta_V}}$$

$$= 2\,\frac{(l_V \text{ Cos } \alpha + z_V \text{ Sin } \alpha)^2}{b^2}\, C_{Y_\beta} \tag{4.62}$$

5. RESULTS AND DISCUSSION

5.1 The methods of estimation of lateral stability derivatives have been applied to a sample case. The Cessna-182 airplane has been considered for this purpose. The mass characteristics of the airplane are given in Table 1. The geometric characteristics and other allied constants which are essential for the estimation of stability derivatives are given in Tables 2 and 3. These constants do not vary with the flight conditions. Table 2 gives the variation of fuselage cross-section area along the body length.

The two flight conditions considered, approach at 73 mph and cruise at 150 mph at 5000 ft. are given in Table 4. Table 5 gives the list of parameters dependent on flight conditions.

Based on the methods of Section 4, lateral stability derivatives have been estimated for the two flight conditions. The values of derivatives along with the contribution of major components as estimated by the methods of Refs.1 to 5 are presented in Table 6(a) and (b). The values estimated from Ref.5 are complete and compare very favourably with the values provided by the manufacturers which are also presented in the same tables. Table 7 makes a comparative study of the estimated derivatives and their values supplied by the

manufacturers, in that, it gives the percentage error from these values.

5.2 In some respects, the methods of estimation given in Refs. 1 to 4 are not complete, in that, all the contributions are not accounted for. A discussion of these methods and values obtained from them follows.

5.2.1 C_{Y_β} : Refs. 1 to 4 do not provide any theoretical relations for estimation of this derivative. All of them suggest use of wind tunnel test data for the design in question. According to Ref.4, interference effects are so large that a generalized formula would not be completely satisfactory. The method of Ref.5, a very recent publication, includes interference effects based on experimental results and hence probably gives the most accurate theoretical estimate.

5.2.2 C_{l_β} : The methods of Refs. 1, 2 and 4 give a slightly higher negative value of wing contribution than that of Ref.5. This is because of the variation in the values of the factors. Only Ref.5 gives a method for estimation of horizontal tail contribution. The vertical tail contribution is almost the same in all cases. The overall value of C_{l_β} from the methods of Refs. 1 to 4 is slightly higher (but compares more favourably with that of the manufactures) and from that of Ref.5 is

5.2.3 <u>C_{n_β}</u> : Ref. 1, 2 and 4 have given relations for estimating wing contribution. This contribution being proportional to square of lift coefficient, is considerable for the approach configuration. If this contribution is considered, the value of C_{n_β} becomes too large compared to manufacturer's value. However, it is negligible for the cruise condition. The other references neglect this contribution entirely. The body contribution from Refs. 1, 2 and 4 is a much higher negative value than that from Ref.5. The method of Ref.3 gives a very low negative value for this contribution. The tail contribution is almost same by all the methods. The overall value of C_{n_β} estimated from Refs. 1 and 4 appears to agree quite well with manufacturer's value for approach configuration only. For the cruise configuration, these two values are too low, The value from Ref. 2 is too low and from Ref.3 is too high for both configurations. The value given by Ref. 5 is more consistent for both the configurations.

5.2.4 <u>C_{Y_p}</u> : As mentioned before, this is not a very important derivative. Refs. 2 and 3 neglect this derivative. The methods of Refs. 1, 4 and 5 give small positive values for this derivative since the angle of attack is more. However, the manufacturers give a negative value for this derivative. The methods of Refs. 1 and 4 take the effects of sidewash into account.

Ref.6 quoting USAF 'DATCOM', gives a method which gives a value of C_{Y_p} much in agreement with manufacturer's value. But since this derivative is not important, the value given by Ref.4 only is considered in the final equations.

5.2.5 $\underline{C_{l_p}}$: Refs. 1, 2 and 5 give the same formula and charts for the wing contribution. The value from Ref.3 also is the same. The method of Ref.4 also gives almost the same value for this contribution. Refs. 1 to 3 do not give any relation for the horizontal tail or vertical tail contributions. The methods of Refs. 4 and 5 give almost the same value for these contributions except that the relation for vertical tail contribution given in Ref.5 is slightly modified in Ref.4. The overall value of C_{l_p} compares well with the manufacturer's value.

5.2.6 $\underline{C_{n_p}}$: The methods of Refs. 1, 2 and 4 give a smaller negative value than that of Ref. 3 or Ref. 5. The tail contribution is almost the same by all the methods. Correspondingly, the overall value of C_{n_p} by methods of Refs.1, 2 and 4 is less negative and that of Refs. 3 and 5 more negative than the manufacturer's value. Ref. 6 gives a formula for $C_{n_{p_W}}$ from NACA-TN 1581 and for $C_{n_{p_V}}$ from NACA-TN 2587 which gives an

overall value which agrees very favourably with manufacturer's value. However, value given by Ref.5 has been used in the solution of equations of motion.

5.2.7 $\underline{C_{Y_r}}$: The value of C_{Y_r} by the method of Ref.1, neglecting the effects of sidewash compare very favourably with manufacturer's values. However, the value by Ref.5 is considered for the solution of equations of motion. Refs. 2 and 3 neglect this derivative entirely.

5.2.8 $\underline{C_{l_r}}$: The value of C_{l_r} by all the methods except Ref.3 are almost same. For the approach configuration this value is as much as 30% more than the manufacturer's value. For the cruise configuration, the agreement is much better. The value by Ref.3 is too rough an estimate.

5.2.9 $\underline{C_{n_r}}$: All the references give the same relation for this derivative. The estimated value compares favourably with the manufacturer's value.

5.3 A consolidated list of estimated derivatives and those supplied by the manufacturers has been given in Table 8. These two sets of values for each of the two flight configurations have been used in the solution of equations of motion

for an initial disturbance of 10^{o} in sideslip angle. Typical input/output for the digital computer programmes of Appendices 'C' and 'E' are shown in Appendix 'F'. The resulting response of the airplane in terms of roll, yaw and sideslip angles and roll and yaw rates are presented in Appendix 'G'.

5.4 Computer programmes were developed for solution of the equations of motion, both by Laplace transform method (Appendices 'A' and 'C') and Fourth order Runge-Kutta method (Appendices 'D' and 'E'). The former method computes the roots of the characteristic equation in addition to the time history of airplane motion, whereas, the latter computes only the time history, but has the advantage of being simpler and quicker. The results obtained by the two methods agree very closely for the time increments used (.05 sec) in the numerical integration. The roots of characteristic equation obtained from the Laplace transform method enable the estimation of the period and damping characteristics. These characteristics for the two flight conditions are given in Table 9(a) and (b).

6. CONCLUSION

6.1 The methods of estimation of lateral stability derivatives from Ref. 5 appear to be more appropriate, since they are complete and the derivatives estimated from these methods for the sample case of Cessna-182H compare very well with those provided by the manufacturers for both approach and cruise configurations.

6.2 The application of the methods of this report to the above sample case predicts dynamically stable lateral airplane motions. In the spiral mode, $T_{1/2}$, the time to damp to half amplitude, is 123 secs. and 62 secs. for the two flight conditions as against 53.5 secs. and 38.5 secs. predicted by the manufacturers values. T his shows a tendency towards spiral unstability which seems to agree with the actual case.

6.3 The variation in $T_{1/2}$ and damping from the manufacturers values for the other two modes, namely, Rolling and Dutch Roll, is within 5% and 10% respectively.

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APPENDIX "A"

FLOW CHART OF SOLUTION OF LATERAL EQUATIONS OF MOTION BY LAPLACE TRANSFORM METHOD

FLOW CHART OF SOLUTION OF LATERAL EQUATIONS OF MOTION BY LAPLACE TRANSFORM METHOD.

START

INPUT: AIRPLANE CHARACTERISTICS & FLIGHT CONDITIONS.

$m, I_{xx}, I_{zz}, \eta, \rho, S, b, V, C_L, \gamma, \epsilon, \alpha$

INPUT: STABILITY DERIVATIVES.

$C_{l_\beta}, C_{n_\beta}, C_{y_\beta}, C_{l_p}, C_{n_p}, C_{y_p}, C_{l_r}, C_{n_r}, C_{y_r}, C_{l_c}, C_{n_c}, C_{y_c}$

INPUT: INITIAL CONDITIONS

$\phi_0, (D\phi)_0, \psi_0, (D\psi)_0, \beta_0$

NON-DIMENSIONALISED PARAMETERS

$$K_{X_0} = \frac{k_{X_0}}{b}$$

$$K_{Z_0} = \frac{k_{Z_0}}{b}$$

$$K_X^2 = K_{X_0}^2 \cos^2\eta + K_{Z_0}^2 \sin^2\eta$$

$$K_Z^2 = K_{Z_0}^2 \cos^2\eta + K_{X_0}^2 \sin^2\eta$$

$$K_{XZ} = (K_{Z_0}^2 - K_{X_0}^2) \sin\eta \cos\eta$$

$$K_1 = \frac{K_{XZ}}{K_X^2}$$

$$K_2 = \frac{K_{XZ}}{K_Z^2}$$

$$\tau = m/\rho S V$$
$$\hat{t} = t/\tau$$
$$\mu = m/\rho S b$$

$$l_\beta = \frac{\mu}{2k_x^2} C_{l_\beta}$$
$$n_\beta = \frac{\mu}{2k_z^2} C_{n_\beta}$$
$$y_\beta = \frac{1}{2} C_{y_\beta}$$
$$l_p = \frac{1}{4k_x^2} C_{l_p}$$
$$n_p = \frac{1}{4k_z^2} C_{n_p}$$
$$y_p = \frac{1}{4\mu} C_{y_p}$$
$$l_r = \frac{1}{4k_x^2} C_{l_r}$$
$$n_r = \frac{1}{4k_z^2} C_{n_r}$$
$$y_r = \frac{1}{4\mu} C_{y_r}$$
$$l_c = \frac{\mu}{2k_x^2} C_{l_c}$$
$$n_c = \frac{\mu}{2k_z^2} C_{n_c}$$
$$y_c = \frac{1}{2} C_{y_c}$$

CALCULATE: CONSTANTS P_1 TO P_7

$$P_1 = -l_p - n_r + K_1 n_p + K_2 l_r$$
$$P_2 = l_p n_r - l_r n_p$$
$$P_3 = l_\beta n_r - l_r n_\beta$$
$$P_4 = l_p n_\beta - l_\beta n_p$$
$$P_5 = K_1 n_\beta - l_\beta$$
$$P_6 = K_2 l_\beta - n_\beta$$
$$P_7 = -P_2 y_\beta + P_3 y_p + P_4 y_r - P_4$$

CALCULAT: COEFFICIENTS OF THE CHARACTERISTIC EQUATION

$$A = 1 - K_1 K_2$$
$$B = P_1 - A\, y_\beta$$
$$C = -P_1 y_\beta + P_2 + P_5 y_p + P_6 y_r - P_6$$
$$D = P_5 \frac{C_L}{2} + P_6 \frac{C_L}{2} \tan\gamma + P_7$$
$$E = P_3 \frac{C_L}{2} + P_4 \frac{C_L}{2} \tan\gamma$$

SOLVE: THE CHARACTERISTIC EQUATION $A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$ BY USING "QUART" SUBROUTINE

$$\lambda_1 = Y(1) + i\,Y(5)$$
$$\lambda_2 = Y(2) + i\,Y(6)$$
$$\lambda_3 = Y(3) + i\,Y(7)$$
$$\lambda_4 = Y(4) + i\,Y(8)$$

CALCULATE: CONSTANTS a_0 to a_5

$a_0 = \phi_0 A$

$a_1 = \phi_0 B + (D\phi)_0 A$

$a_2 = \phi_0 C + \beta_0 P_5 + (D\phi)_0 (-A y_p + K_2 l_r - n_r) - (D\psi)_0 (K_1 n_r - l_r) + l_c - n_c K_1$

$a_3 = \phi_0 (P_6 \frac{C_L}{2} \tan\nu + P_7) - \psi_0 (P_5 \frac{C_L}{2} \tan\nu) - \beta_0 P_3 + (D\phi)_0 (P_6 y_r - P_6 - K_2 l_r y_\beta + n_r y_\beta) + (D\psi)_0 (-P_5 y_r + P_5 + K_1 n_r y_\beta - l_r y_\beta) - l_c (n_r + y_\beta) - y_c P_5$

$a_4 = [\phi_0 P_4 - \psi_0 P_3 + (D\phi)_0 P_6 - (D\psi)_0 P_5] \frac{C_L}{2} \tan\nu + l_c (n_\beta - n_\beta y_r + n_r y_\beta) + n_c (l_\beta y_r - l_\beta - l_r y_\beta) - y_c P_3$

$a_5 = (-l_c n_\beta + n_c l_\beta) \frac{C_L}{2} \tan\nu$

CALCULATE: CONSTANTS b_0 to b_5

$b_0 = \psi_0 A$

$b_1 = \psi_0 B + (D\psi)_0 A$

$b_2 = \psi_0 C - \beta_0 P_6 - (D\phi)_0 (K_2 l_p - n_p) + (D\psi)_0 (-A y_\beta + K_1 n_p - l_p) - l_c K_2 + n_c$

$b_3 = -\phi_0 P_6 \frac{C_L}{2} + \psi_0 (P_5 \frac{C_L}{2} + P_7) - \beta_0 P_4 + (D\phi)_0 (-P_6 y_\beta + K_2 l_p y_\beta - n_p y_\beta) + (D\psi)_0 (-P_5 y_p - K_1 n_p y_\beta + l_p y_\beta) - l_c (K_2 y_\beta + n_p) - n_c (l_p + y_\beta) - y_c P_6$

$b_4 = [-\phi_0 P_4 + \psi_0 P_3 - (D\phi)_0 P_6 + (D\psi)_0 P_5] \frac{C_L}{2} + l_c (n_\beta y_p - n_p y_\beta) + n_c (l_p y_\beta - l_\beta y_p) - y_c P_4$

$b_5 = (l_c n_\beta - l_\beta n_c) \frac{C_L}{2}$

CALCULATE: CONSTANTS C_0 to C_4

$C_0 = \beta_0 A$

$C_1 = \phi_0 A \frac{C_L}{2} + \psi_0 A \frac{C_L}{2}\tan\nu + \beta_0 P_1 + (D\phi)_0 A y_p - (D\psi)_0 A (y_r - 1) + y_c A$

$C_2 = \phi_0 P_1 \frac{C_L}{2} + \psi_0 P_1 \frac{C_L}{2}\tan\nu + \beta_0 P_2 + (D\phi)_0 \left[A \frac{C_L}{2} - K_2 l_p y_r + K_2 l_p + n_p y_r - n_p + (K_2 l_r - n_r) y_p\right] + (D\psi)_0 \left[A \frac{C_L}{2}\tan\nu + K_1 n_p y_r - K_1 n_p - l_p y_r - (K_1 n_r - l_r) y_p\right] + l_c(-K_2 y_r + K_2 + y_p) + n_c(y_r - 1 - K_1 y_p) + y_c P_1$

$C_3 = \phi_0 P_2 \frac{C_L}{2} + \psi_0 P_2 \frac{C_L}{2}\tan\nu + (D\phi)_0 \left[-K_2 l_p \frac{C_L}{2}\tan\nu + n_p \frac{C_L}{2}\tan\nu + K_2 l_r \frac{C_L}{2} - n_r \frac{C_L}{2}\right] + (D\psi)_0 \left[K_1 n_p \frac{C_L}{2}\tan\nu - l_p \frac{C_L}{2}\tan\nu - K_1 n_r \frac{C_L}{2} + l_r \frac{C_L}{2}\right] + l_c\left(n_p y_r - n_p - n_r y_p + \frac{C_L}{2} - K_2 \frac{C_L}{2}\tan\nu\right) + n_c\left(-l_p y_r + l_p + l_r y_p - K_1 \frac{C_L}{2} + \frac{C_L}{2}\tan\nu\right) + y_c P_2$

$C_4 = l_c\left(n_p \frac{C_L}{2}\tan\nu - n_r \frac{C_L}{2}\right) + n_c\left(l_r \frac{C_L}{2} - l_p \frac{C_L}{2}\tan\nu\right)$

CALCULATE: CONSTANTS A_1 to A_6, B_1 to B_6, C_1 to C_5 and E_1 to E_4

$A_5 = \frac{a_5}{E}$

$A_6 = \frac{1}{E}\left(a_4 - a_5 \frac{D}{E}\right)$

$B_5 = \frac{b_5}{E}$

$B_6 = \frac{1}{E}\left(b_4 - \frac{b_5 D}{E}\right)$

$C_5 = \frac{C_4}{E}$

(COMMON TO 5, 15 AND 15)

$Y(5)$ $\xrightarrow{\neq 0}$ 15 → (P. 65) (AT LEAST ONE PAIR OF COMPLEX ROOTS)

$= 0$

$Y(7)$ $\xrightarrow{\neq 0}$ 115 → (P. 68) (ONE PAIR OF COMPLEX ROOTS)

$= 0$ (NO COMPLEX ROOTS)

5

$$E_i = (6A\lambda_i^5 + 5B\lambda_i^4 + 4C\lambda_i^3 + 3D\lambda_i^2 + 2E\lambda_i) \; ; \; i = 1,4$$

$$A_i = \frac{\sum_{n=0}^{5} a_n \lambda_i^{5-n}}{E_i} \; ; \; i = 1,4$$

$$B_i = \frac{\sum_{n=0}^{5} b_n \lambda_i^{5-n}}{E_i} \; ; \; i = 1,4$$

$$C_i = \frac{\sum_{n=0}^{4} c_n \lambda_i^{5-n}}{E_i} \; ; \; i = 1,4$$

CALCULATE: FLIGHT PATH PARAMETERS

$$\phi = A_1 e^{\sigma\lambda_1} + A_2 e^{\sigma\lambda_2} + A_3 e^{\sigma\lambda_3} + A_4 e^{\sigma\lambda_4} + A_5\sigma + A_6$$

$$\psi = B_1 e^{\sigma\lambda_1} + B_2 e^{\sigma\lambda_2} + B_3 e^{\sigma\lambda_3} + B_4 e^{\sigma\lambda_4} + B_5\sigma + B_6$$

$$\beta = C_1 e^{\sigma\lambda_1} + C_2 e^{\sigma\lambda_2} + C_3 e^{\sigma\lambda_3} + C_4 e^{\sigma\lambda_4} + C_5$$

$$p = \frac{1}{\tau}\left(A_1\lambda_1 e^{\sigma\lambda_1} + A_2\lambda_2 e^{\sigma\lambda_2} + A_3\lambda_3 e^{\sigma\lambda_3} + A_4\lambda_4 e^{\sigma\lambda_4} + A_5\right)$$

$$r = \frac{1}{\tau}\left(B_1\lambda_1 e^{\sigma\lambda_1} + B_2\lambda_2 e^{\sigma\lambda_2} + B_3\lambda_3 e^{\sigma\lambda_3} + B_4\lambda_4 e^{\sigma\lambda_4} + B_5\right)$$

OUTPUT

ϕ, ψ, β, p, r

(65)

STOP

(P.64)

(15)

$$A_1 = \frac{\sum_{n=0}^{5} a_n \lambda_1^{5-n}}{(6A\lambda_1^5 + 5B\lambda_1^4 + 4C\lambda_1^3 + 3D\lambda_1^2 + 2E\lambda_1)} = R_A + iI_A$$

(SEE APPX 'B')

$$A_2 = \frac{\sum_{n=0}^{5} a_n \lambda_2^{5-n}}{(6A\lambda_2^5 + 5B\lambda_2^4 + 4C\lambda_2^3 + 3D\lambda_2^2 + 2E\lambda_2)} = R_A - iI_A$$

$$K_A = 2\sqrt{R_A^2 + I_A^2}$$

$$\omega_A = \tan^{-1}\left(\frac{I_A}{R_A}\right)$$

$$B_1 = \frac{\sum_{n=0}^{5} b_n \lambda_1^{5-n}}{(6A\lambda_1^5 + 5B\lambda_1^4 + 4C\lambda_1^3 + 3D\lambda_1^2 + 2E\lambda_1)} = R_B + i\,I_B$$

$$B_2 = \frac{\sum_{n=0}^{5} b_n \lambda_2^{5-n}}{(6A\lambda_2^5 + 5B\lambda_2^4 + 4C\lambda_2^3 + 3D\lambda_2^2 + 2E\lambda_2)} = R_B - i\,I_B$$

$$K_B = 2\sqrt{R_B^2 + I_B^2}$$

$$\omega_B = \tan^{-1}\left(\frac{I_B}{R_B}\right)$$

$$C_1 = \frac{\sum_{n=0}^{4} c_n \lambda_1^{5-n}}{(6A\lambda_1^5 + 5B\lambda_1^4 + 4C\lambda_1^3 + 3D\lambda_1^2 + 2E\lambda_1)} = R_C + i\,I_C$$

$$C_2 = \frac{\sum_{n=0}^{4} c_n \lambda_2^{5-n}}{(6A\lambda_2^5 + 5B\lambda_2^4 + 4C\lambda_2^3 + 3D\lambda_2^2 + 2E\lambda_2)} = R_C - i\,I_C$$

$$K_C = 2\sqrt{R_C^2 + I_C^2}$$

$$\omega_C = \tan^{-1}\left(\frac{I_C}{R_C}\right)$$

$y(7)$

$\neq 0$ → (125) → (p.68)

$= 0$ → (25)

$$E_i = (6A\lambda_i^5 + 5B\lambda_i^4 + 4C\lambda_i^3 + 3D\lambda_i^2 + 2E\lambda_i) \; ; \; i = 3,4$$

$$A_i = \frac{\sum_{n=0}^{5} a_n \lambda_i^{5-n}}{E_i} \; ; \; i = 3,4$$

$$B_i = \frac{\sum_{n=0}^{5} b_n \lambda_i^{5-n}}{E_i} \; ; \; i = 3,4$$

$$C_i = \frac{\sum_{n=0}^{4} c_n \lambda_i^{5-n}}{E_i} \; ; \; i = 3,4$$

CALCULATE: FLIGHT PATH PARAMETERS

$$\phi = K_A e^{\sigma R} \cos(\sigma I + \omega_A) + A_3 e^{\sigma\lambda_3} + A_4 e^{\sigma\lambda_4} + A_5\sigma + A_6$$

$$\psi = K_B e^{\sigma R} \cos(\sigma I + \omega_B) + B_3 e^{\sigma\lambda_3} + B_4 e^{\sigma\lambda_4} + B_5\sigma + B_6$$

$$\beta = K_C e^{\sigma R} \cos(\sigma I + \omega_C) + C_3 e^{\sigma\lambda_3} + C_4 e^{\sigma\lambda_4} + C_5$$

$$p = \frac{1}{\tau}\left[K_A\sqrt{R^2+I^2}\, e^{\sigma R} \cos\left(\sigma I + \omega_A + \tan^{-1}\left(\frac{I}{R}\right)\right) + A_5 + A_3\lambda_3 e^{\sigma\lambda_3} + A_4\lambda_4 e^{\sigma\lambda_4}\right]$$

$$r = \frac{1}{\tau}\left[K_B\sqrt{R^2+I^2}\, e^{\sigma R} \cos\left(\sigma I + \omega_B + \tan^{-1}\left(\frac{I}{R}\right)\right) + B_5 + B_3\lambda_3 e^{\sigma\lambda_3} + B_4\lambda_4 e^{\sigma\lambda_4}\right]$$

OUTPUT
ϕ, ψ, β, p, r

(65)

STOP

115 (p.64)

$$E_i = \left(6A\lambda_i^5 + 5B\lambda_i^4 + 4C\lambda_i^3 + 3D\lambda_i^2 + 2E\lambda_i\right), \; i = 1,2$$

$$A_i = \frac{\sum_{n=0}^{5} a_n \lambda_i^{5-n}}{E_i} \; , \; i = 1,2$$

$$B_i = \frac{\sum_{n=0}^{5} b_n \lambda_i^{5-n}}{E_i} \; ; \; i = 1,2$$

$$C_i = \frac{\sum_{n=0}^{4} c_n \lambda_i^{5-n}}{E_i} \; ; \; i = 1,2$$

(p.66) → 125

$$\lambda_3 = Y(3) + i\,Y(7) = R' + iI'$$

$$\lambda_4 = Y(4) + i\,Y(8) = R' - iI'$$

$$A_3 = \frac{\sum_{n=0}^{5} a_n \lambda_3^{5-n}}{\left(6A\lambda_3^5 + 5B\lambda_3^4 + 4C\lambda_3^3 + 3D\lambda_3^2 + 2E\lambda_3\right)} = R'_A + iI'_A$$

(SEE APPENDIX 'B')

$$A_4 = \frac{\sum_{n=0}^{5} a_n \lambda_4^{5-n}}{\left(6A\lambda_4^5 + 5B\lambda_4^4 + 4C\lambda_4^3 + 3D\lambda_4^2 + 2E\lambda_4\right)} = R'_A - iI'_A$$

$$K'_A = 2\sqrt{R'^2_A + I'^2_A}$$

$$w'_A = \tan^{-1}\left(\frac{I'_A}{R'_A}\right)$$

$$B_3 = \frac{\sum_{n=0}^{5} b_n \lambda_3^{(5-n)}}{(6A\lambda_3^5 + 5B\lambda_3^4 + 4C\lambda_3^3 + 3D\lambda_3^2 + 2E\lambda_3)} = R'_B + i I'_B$$

$$B_4 = \frac{\sum_{n=0}^{5} b_n \lambda_4^{5-n}}{(6A\lambda_4^5 + 5B\lambda_4^4 + 4C\lambda_4^3 + 3D\lambda_4^2 + 2E\lambda_4)} = R'_B - i I'_B$$

$$K'_B = 2\sqrt{R'^2_B + I'^2_B}$$

$$\omega'_B = \tan^{-1}\left(\frac{I'_B}{R'_B}\right)$$

$$C_3 = \frac{\sum_{n=0}^{4} c_n \lambda_3^{5-n}}{(6A\lambda_3^5 + 5B\lambda_3^4 + 4C\lambda_3^3 + 3D\lambda_3^2 + 2E\lambda_3)} = R'_C + i I'_C$$

$$C_4 = \frac{\sum_{n=0}^{4} c_n \lambda_4^{5-n}}{(6A\lambda_4^5 + 5B\lambda_4^4 + 4C\lambda_4^3 + 3D\lambda_4^2 + 2E\lambda_4)} = R'_C - i I'_C$$

$$K'_C = 2\sqrt{R'^2_C + I'^2_C}$$

$$\omega'_C = \tan^{-1}\left(\frac{I'_C}{R'_C}\right)$$

$Y(5)$ $\xrightarrow{=0}$ (650) $\rightarrow$ (p. 70)

$\neq 0$ $\downarrow$ (6)

CALCULATE: FLIGHT PATH PARAMETERS.

$$\phi = K_A e^{\sigma R} \cos(\sigma I + \omega_A) + K_A' e^{\sigma R'} \cos(\sigma I' + \omega_A') + A_5\sigma + A_6$$

$$\psi = K_B e^{\sigma R} \cos(\sigma I + \omega_B) + K_B' e^{\sigma R'} \cos(\sigma I' + \omega_B') + B_5\sigma + B_6$$

$$\beta = K_C e^{\sigma R} \cos(\sigma I + \omega_C) + K_C' e^{\sigma R'} \cos(\sigma I' + \omega_C') + C_5$$

$$p = \frac{1}{\tau}\left[K_A \sqrt{R^2 + I^2} \cos\left(\sigma I + \omega_A + \tan^{-1}\frac{I}{R}\right) + A_5 + K_A' \sqrt{R'^2 + I'^2} \cos\left(\sigma I' + \omega_A' + \tan^{-1}\frac{I'}{R'}\right)\right]$$

$$r = \frac{1}{\tau}\left[K_B \sqrt{R^2 + I^2} \cos\left(\sigma I + \omega_B + \tan^{-1}\frac{I}{R}\right) + B_5 + K_B' \sqrt{R'^2 + I'^2} \cos\left(\sigma I' + \omega_B' + \tan^{-1}\frac{I'}{R'}\right)\right]$$

OUTPUT

ϕ, ψ, β, p, r

65

STOP

(p. 69)

650

CALCULATE: FLIGHT PATH PARAMETERS

$$\phi = A_1 e^{\sigma\lambda_1} + A_2 e^{\sigma\lambda_2} + K_A' e^{\sigma R'} \cos(\sigma I' + \omega_A') + A_5\sigma + A_6$$

$$\psi = B_1 e^{\sigma\lambda_1} + B_2 e^{\sigma\lambda_2} + K_B' e^{\sigma R'} \cos(\sigma I' + \omega_B') + B_5\sigma + B_6$$

$$\beta = C_1 e^{\sigma\lambda_1} + C_2 e^{\sigma\lambda_2} + K_C' e^{\sigma R'} \cos(\sigma I' + \omega_C') + C_5$$

$$p = \frac{1}{\tau}\left[A_1\lambda_1 e^{\sigma\lambda_1} + A_2\lambda_2 e^{\sigma\lambda_2} + A_5 + K_A' \sqrt{R'^2 + I'^2} \cos\left(\sigma I' + \omega_A' + \tan^{-1}\frac{I'}{R'}\right)\right]$$

$$r = \frac{1}{\tau}\left[B_1\lambda_1 e^{\sigma\lambda_1} + B_2\lambda_2 e^{\sigma\lambda_2} + B_5 + K_B' \sqrt{R'^2 + I'^2} \cos\left(\sigma I' + \omega_B' + \tan^{-1}\frac{I'}{R'}\right)\right]$$

OUTPUT

ϕ, ψ, β, p, r

65

STOP

APPENDIX 'B'

SPECIAL NOTATIONS USED IN SECTION 3

SPECIAL NOTATIONS USED IN SECTION 3

If two roots λ_1, λ_2 of the characteristic equation are complex, then we can write

$$\lambda_1, \lambda_2 = R \pm i\, I \qquad (B.1)$$

Further, we can write the power of λ_1 as follows

$$\lambda_1^k = (R + i\, I)^k$$

$$= R^k + kR^{k-1}(iI) + \frac{k.(k-1)}{2!} R^{k-2} (iI)^2 \ldots .+ (iI)^k$$

$$= R^k - k\frac{(k-1)}{2!} R^{k-2} I^2 + \frac{k.(k-1).(k-2)-(k-3)}{4!} R^{(k-4)} I^4 -$$

$$+ i(kR^{k-1} I - \frac{k.(k-1).(k-2)}{3!} R^{k-3} I^3 + \ldots\ldots)$$

$$= R_k + i\, I_k \qquad (B.2)$$

We can now substitute for λ_1 in the expression for the coefficient A_1

$$A_1 = \frac{\sum_{j=0}^{4} a_j (R_{5-j} + iI_{5-j}) + a_5}{(6AR_5 + 5BR_4 + 4CR_3 + 3DR_2 + 2ER_1) + i(6AI_5 + 5BI_4 + 4CI_3 + 3DI_2 + 2EI_1)}$$

which can be written as

$$A_1 = \frac{x_1 + i\, y_1}{x_2 + i\, y_2} = \left(\frac{x_1\, x_2 + y_1\, y_2}{x_2^2 + y_2^2}\right) + i\left(\frac{x_2 y_1 - x_1 y_2}{x_2^2 + y_2^2}\right)$$

$$= R_A + i\, I_A \qquad \text{(B.4)}$$

A_2 will be the complex conjugate of A_1 and will be

$$A_2 = R_A - i\, I_A \qquad \text{(B.5)}$$

Similarly

$$B_1,\ B_2 = R_B \pm i\, I_B \qquad \text{(B.6)}$$

$$C_1,\ C_2 = R_C \pm i\, I_C \qquad \text{(B.7)}$$

If the other pair of roots is also complex, then exactly as above

$$A_3,\ A_4 = R'_A \pm i\, I'_A \qquad \text{(B.8)}$$

$$B_3,\ B_4 = R'_B \pm i\, I'_B \qquad \text{(B.9)}$$

$$C_3,\ C_4 = R'_C \pm i\, I'_C \qquad \text{(B.10)}$$

- - -

APPENDIX 'C'

DIGITAL COMPUTER PROGRAMME LISTING FOR SOLUTION OF LATERAL EQUATIONS OF MOTION BY LAPLACE TRANSFORM METHOD

```
C **** THIS PROGRAMME EVALUATES THE COEFFICIENTS OF CHARACTERISTIC EQUA-
C **** TION OF LATERAL MOTION OF AN AIRPLANE USING MASS, GEOMETRIC AND
C **** AERODYNAMIC PARAMETERS, SOLVES THE QUARTIC EQUATON, SOLVES THE
C **** EQUATIONS OF MOTION BY LAPLACE TRANSFORM METHOD TO PREDICT THE
C **** TIME HISTORY OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL AND YAW
C **** RATES.
      DIMENSIONPHID(200),PSID(200),BETAD(200),ROVELD(200),YAVELD(200),
     1T1(200)
      DIMENSION TIME(5,120),PARAM(5,120),NP(5)
      COMMON/GPHM/ISCALE,NPLOT,NOPRNT,IHSIZE,JVSIZE
      NPLOT=1
      IHSIZE=12
      JVSIZE=8
      ISCALE=0
      NOPRNT=1
      DIMENSION AAY(10),BBY(10)
      DIMENSION AA(10),BB(10),CC(10),        AD(10),E(10),BD(10),CD(10)
      DIMENSION A(10)
      DIMENSION PER(2),THALF(4)
      READ 2101,WEIT,XIKO,ZIKO,SPAN,AREA
      READ 2501,PHIO,DPHIO,PSIO,DPSIO,BTDO
      DO 165 L=1,2
      READ 2101,VELM,RHO,EPSD,ALFAD,GAMMAD
      DO 165 LJK=1,2
      READ 2501,CLB,CNB,CYB
      READ 2501,CLP,CNP,CYP
      READ 2501,CLR,CNR,CYR
      READ 2501,CLC,CNC,CYC
      GRTY=32.2
      PI=4.*ATAN(1.)
      PID=180./PI
      AMASS=WEIT/GRTY
      ETA=(ALFAD-EPSD)/PID
      GAMMA=GAMMAD/PID
      BTAO=BTDO/PID
      VEL=VELM*88./60.
      CL=2.*WEIT/(RHO*VEL**2*AREA)
      XKO=SQRT(XIKO/AMASS)
      ZKO=SQRT(ZIKO/AMASS)
      CLG=.5*CL*TAN(GAMMA)
      XXK=((XKO*COS(ETA))**2+(ZKO*SIN(ETA))**2)/(SPAN**2)
      ZZK=((ZKO*COS(ETA))**2+(XKO*SIN(ETA))**2)/(SPAN**2)
      XZK=((ZKO**2-XKO**2)*SIN(ETA)*COS(ETA))/(SPAN**2)
      AK1=XZK/XXK
      AK2=XZK/ZZK
      TOU=AMASS/(RHO*AREA*VEL)
      PERT=2.*PI*TOU
      THN=-0.693*TOU
      AMU=AMASS/(RHO*AREA*SPAN)
      SLB=.5*AMU/XXK*CLB
      SLP=0.25*CLP/XXK
      SLR=0.25*CLR/XXK
      SLC=.5*AMU/XXK*CLC
      SNB=.5*AMU/ZZK*CNB
      SNP=0.25*CNP/ZZK
```

```
      SNR=0.25*CNR/ZZK
      SNC=.5*AMU/ZZK*CNC
      YB=.5 *CYB
      YP=.25*CYP/AMU
      YR=.25*CYR/AMU
      YC=.5*CYC
      P1=-SLP-SNR+AK1*SNP+AK2*SLR
      P2=SLP*SNR-SLR*SNP
      P3=SLB*SNR-SLR*SNB
      P4=SLP*SNB-SLB*SNP
      P5=AK1*SNB-SLB
      P6=AK2*SLB-SNB
      P7=-P2*YB+P3*YP+P4*YR-P4
      DIMENSION  Y(8)
      A(2)=1.-AK1*AK2
      A(3)=P1-A(2)*YB
      A(4)=-P1*YB+P2+P5*YP+P6*YR-P6
      A(5)=P5*.5*CL+P6*CLG+P7
      A(6)=P3*.5*CL+P4*CLG
      PRINT 3C[illegible]
      PRINT 3C01,WEIT,XIKO,ZIKO,SPAN,AREA
      PRINT 3C03,VFLM,RHO,CL ,ALFAD,GAMMAD
      PRINT 3C[illegible],CYB,CYP,CYR,CYC,CLB,CLP,CLR,CLC,CNB,CNP,CNR,CNC
      PRINT 3C[illegible],PHI0,DPHI0,PSI0,DPSI0,BTD0
      PRINT 3C05
      PRINT 3C2[illegible]
      PRINT  3[illegible]1
      PRINT  3[illegible]12
      PRINT 3C0[illegible]
      PRINT  3027
      PRINT 40,(A(I),I=2,6)
      A0=PHI0*A(2)
      A1=PHI0*A(3)+DPHI0*A(2)
      A2=PHI0*A(4)-BTA0*P5+DPHI0*(-A(2)*YB+AK2*SLR-SNR)
     1-DPSI0*(AK1*SNR-SLR)+SLC-SNC*AK1
      A3=PHI0*(P6*CLG+P7)- PSI0*P5*CLG-BTA0*P3+DPHI0*(P6*YR-P6-AK2*
     1SLR*YB)+DPSI0*(-P5*YR+P5+AK1*SNR*YB-SLR*YB)-SLC*(SNR+YB)+
     2SNC*(AK1*YB+SLR)-YC*P5+DPHI0*SNR*YB
      A4=(PHI0*P4-PSI0*P3+DPHI0*P6-DPSI0*P5)*CLG+SLC*(SNB-SNB*
     1YR+SNR*YB)+SNC*(SLB*YR-SLB-SLR*YB)-YC*P3
      A5=CLG*(-SLC*SNB+SNC*SLB)
      B0=PSI0*A(2)
      B1=PSI0*A(3)+DPSI0*A(2)
      B2=PSI0*A(4)-BTA0*P6-DPHI0*(AK2*SLP-SNP)+DPSI0*(-A(2)*YB+AK1*
     1SNP-SLP)-SLC*AK2+SNC
      B3=-.5*PHI0*P6*CL+PSI0*(.5*P5*CL+P7)-BTA0*P4+DPHI0*(-P6*YP+
     1AK2*SLP*YB-SNP*YB)+DPSI0*(P5*YP-AK1*SNP*YB+SLP*YB)+
     2SLC*(AK2*YB+SNP)-SNC*(SLP+YB)- YC*P6
      B4=.5*(          PSI0*P3-DPHI0*P6+DPSI0*P5)*CL+SLC*(SNB*YP-SNP*YB)
     1+SNC*(SLP*YB-SLB*YP)-YC*P4-0.5*PHI0*P4*CL
      B5=.5*(SLC*SNB-SNC*SLB)*CL
      C0=BTA0*A(2)
      C1=.5*PHI0*A(2)*CL+PSI0*A(2)*CLG+BTA0*P1+DPHI0*A(2)*YP-DPSI0*
     1A(2)*(YR-[illegible])+YC*A(2)
      C2=.5*PHI0*P1*CL+PSI0*P1*CLG+BTA0*P2+DPHI0*(.5*CL*A(2)-AK2*SLP*
```

```
     4YR+AK2*SLP
     1+SNP*YR-SNP+(AK2*SLR-SNR)*YP)+DPSIO*(A(2)*CLG+AK1*SNP*YR-
     2AK1*SNP-SLP*YR+SLP-(AK1*SNR-SLR)*YP)+SLC*(-AK2*YR+AK2+YP)+SNC*
     3(YR-1.-AK1*YP)+YC*P1
      C3=            PSIO*P2*CLG+DPHIO*(-AK2*SLP*CLG+SNP*CLG+AK2*SLR*
     1CL*.5-SNR*CL*.5)+DPSIO*(AK1*SNP*CLG-SLP*CLG-AK1*SNR*CL*.5+SLR*CL*
     2.5)+SLC*(SNP*YR-SNP-SNR*YP+.5*CL-AK2*CLG)+SNC*(-SLP*YR+SLP+
     3SLR*YP-AK1*CL*.5+CLG)+YC*P2+0.5*CL*PHIO*P2
      C4=SLC*(SNP*CLG-SNR*.5*CL)+SNC*(SLR*CL*.5-SLP*CLG)
      CALL QUART(A,Y)
      DO 1 I=1,4
1     THALF(I)=THN/Y(I)
      IF (Y(5). Q.0.0) GO TO 2
      PER(1)=PERT/ABS(Y(5))
2     IF(Y(7).EQ.0.0)  GO TO 3
      PER(2)=PERT/ABS(Y(7))
3     CONTINUE
      PRINT 305 ,(PER(I),I=1,2),(THALF(I),I=1,4)
3051  FORMAT(/5X,6(F20.4))
      AA(5)=A5/A(6)
      AA(6)=(A4-A5*A(5)/A(6))/A(6)
      BB(5)=B5/A(6)
      BB(6)=(B4-B5*A(5)/A(6))/A(6)
      CC(5)=C4/A(6)
      AAY(5)=AA(5)/TOU
      BBY(5)=BB(5)/TOU
      IF(Y(5).NE.0.0)  GO TO 15
      IF(Y(7).NE.0.0)  GO TO    115
      DO 20 I=1,4
      AD(I)=A5+Y(I)*(A4+Y(I) *(A3+Y(I)*(A2+Y(I)*(A1+Y(I)*A0))))
      E(I)=Y(I)*(2.*A(6)+Y(I)*(3.*A(5)+Y(I)*(4.*A(4)+Y(I)*(5.*A(3)+
     1Y(I)*6.*A(2)))))
      AA(I)=AD(I)/E(I)
      BD(I)=B5+Y(I)*(B4+Y(I)*(B3+Y(I)*(B2+Y(I)*(B1+Y(I)*B0))))
      BB(I)=BD(I)/E(I)
      CD(I)=Y(I)*(C4+Y(I)*(C3+Y(I)*(C2+Y(I)*(C1+Y(I)*C0))))
      CC(I)=CD(I)/E(I)
      AAY(I)=AA(I)*Y(I)/TOU
      BBY(I)=BB(I)*Y(I)/TOU
20    CONTINUE
      PRINT  3000
3000  FORMAT(//10X,*AA(1)=*,11X,*AA(2)=*,11X,*AA(3)=*,11X,*AA(4)=*,
     111X,*AA(5)=*,11X,*AA(6)=*)
      PRINT  3050,(AA(I),I=1,6)
3050  FORMAT(/4X,6(4X,F8.5,4X))
      PRINT  3100
3100  FORMAT(//10X,*BB(1)=*,11X,*BB(2)=*,11X,*BB(3)=*,11X,*BB(4)=*,
     1 11X,*BB(5)=*,11X,*BB(6)=*)
      PRINT  3050,(BB(I),I=1,6)
      PRINT  3200
3200  FORMAT(//10X,*CC(1)=*,11X,*CC(2)=*,11X,*CC(3)=*,11X,*CC(4)=*,
     111X,*CC(5)=*)
      PRINT  3050,(CC(I),I=1,5)
      PRINT  3300
3300  FORMAT(//10X,*AAY(1)=*,11X,*AAY(2)=*,11X,*AAY(3)=*,11X,
```

```
     1*AAY(4)=*,11X,*AAY(5)=*)
      PRINT   3050,(AAY(I),I=1,5)
      PRINT    3400
3400  FORMAT(//10X,*BBY(1)=*,11X,*BBY(2)=*,11X,*BBY(3)=*,
     111X,*BBY(4)=*,11X,*BBY(5)=*)
      PRINT   3050,(BBY(I),I=1,5)
      PRINT 3027
      PRINT 3C0
      PRINT 3C25
      PRINT 3C0
      PRINT   3027
      PRINT   80,(AA(I),Y(I),I=1,4),AA(5),AA(6)
80    FORMAT(//5X,*..........=*,4(F10.6,*EXP(SIGMA*,F10.6,*)+*)/
     15X,F10.6,*SIGMA+*,F10.6)
      PRINT   80,(BB(I),Y(I),I=1,4),BB(5),BB(6)
      PRINT   90,(CC(I),Y(I),I=1,4),CC(5)
      PRINT   90,(AAY(I),Y(I),I=1,4),AAY(5)
      PRINT   90,(BBY(I),Y(I),I=1,4),BBY(5)
90    FORMAT(//5X,*..........=*,4(F10.6,*EXP(SIGMA*,F10.6,*)+*)/
     15X,F10.6)
1015  T=0.0
      DO 100  LIKE=1,120
      DIMENSION  AB(10),BA(10),BC(10),BAAY(10),ABBY(10),ZD(10)
      SIGMA=T/TAU
      SUM =0.
      SUMP=0.0
      SMP =0.0
      SMUP =0.0
      TOT=0.0
      DO  1010 I=1,4
      P=SIGMA*Y(I)
      ZD(I)=EXP(P)
      BA(I)=AA(I)*ZD(I)
      BAAY(I)=AAY(I)*ZD(I)
      AB(I)=BB(I)*ZD(I)
      ABBY(I)=BBY(I)*ZD(I)
      BC(I)=CC(I)*ZD(I)
      SUM=SUM+BA(I)
      SUMP=SUMP+BAAY(I)
      SMP=SMP+AB(I)
      SMUP=SMUP+ABBY(I)
      TOT=TOT+BC(I)
1010  CONTINUE
      PHI=SUM+AA(5)*SIGMA+AA(6)
      PSI=SMP+BB(5)*SIGMA+BB(6)
      BETA=TOT+CC(5)
      ROLVEL=SUMP+AAY(5)
      YAWVEL=SMUP+BBY(5)
      PHID(LIKE)=PHI*180./PI
      PSID(LIKE)=PSI*180./PI
      BETAD(LIKE)=BETA*180./PI
      ROVELD(LIKE)=ROLVEL*180./PI
      YAVELD(LIKE)=YAWVEL*180./PI
      T1(LIKE)=T
      T=T+0.05
```

```
1000 CONTINUE
     GO TO 65
 15  R1=Y(1)
     R2=Y(1)**2-Y(5)**2
     R3=Y(1)**3-3.*Y(1)*Y(5)**2
     R4=Y(1)**4-6.*(Y(1)*Y(5))**2+Y(5)**4
     R5=Y(1)**5-10.*Y(1)**3*Y(5)**2+5.*Y(1)*Y(5)**4
     AI1=Y(5)
     AI2=2.*Y(5)*Y(1)
     AI3=3.*Y(1)**2*Y(5)-Y(5)**3
     AI4=4.*(Y(1)**3*Y(5)-Y(1)*Y(5)**3)
     AI5=(5.*Y(1)**4-10.*(Y(1)*Y(5))**2+Y(5)**4)*Y(5)
     AX1=A0*R5+A1*R4+A2*R3+A3*R2+A4*R1+A5
     AY1=A0*AI5+A1*AI4+A2*AI3+A3*AI2+A4*AI1
     X2=6.*A(2)* R5+5.*A(3)* R4+4.*A(4)* R3+3.*A(5)*  R2+2.*A(6)*  R1
     Y2=6.*A(2)*AI5+5.*A(3)*AI4+4.*A(4)*AI3+3.*A(5)*AI2+2.*A(6)*AI1
     Z=X2**2 +Y2**2
     RA  =(AX1*X2+AY1*Y2)/Z
     AIA  =(X2*AY1-AX1*Y2)/Z
     BX1=B0*R5+B1*R4+B2*R3+B3*R2+B4*R1+B5
     BY1=B0*AI5+B1*AI4+B2*AI3+B3*AI2+B4*AI1
     RB  =(BX1*X2+BY1*Y2)/Z
     BIB  =(X2*BY1-BX1*Y2)/Z
     CX1=C0*R5+C1*R4+C2*R3+ C3*R2+C4*R1
     CY1=C0*AI5+C1*AI4+C2*AI3+C3*AI2+C4*AI1
     RC  =(CX1*X2+CY1*Y2)/Z
     CIC  =(X2*CY1-CX1*Y2)/Z
     AKA=2.*SQRT(RA**2+AIA**2)
     BKB=2.*SQRT(RB**2+BIB**2)
     CKC=2.*SQRT(RC**2+CIC**2)
     CALL  ARCTAN(RA  ,AIA  ,WA  )
     CALL  ARCTAN(RB,  BIB,  WB)
     CALL  ARCTAN(RC  ,CIC  ,WC  )
     CALL  ARCTAN(Y(1),Y(5),WAA)
     YAKA=SQRT(Y(1)**2+Y(5)**2)/TOU
     AKAA=AKA*YAKA
     AKAB=BKB*YAKA
     IF(Y(7).NE.0.0)   GO TO 125
 25  DO 50 I=3,4
     AD(I)=A5+Y(I)*(A4+Y(I)  *(A3+Y(I)*(A2+Y(I)*(A1+Y(I)*A0)))))
     E(I)=Y(I)*(2.*A(6)+Y(I)*(3.*A(5)+Y(I)*(4.*A(4)+Y(I)*(5.*A(3)+
    1Y(I)*6.*A(2))))))
     AA(I)=AD(I)/E(I)
     BD(I)=B5+Y(I)*(B4+Y(I)*(B3+Y(I)*(B2+Y(I)*(B1+Y(I)*B0)))))
     CD(I)=Y(I)*(C4+Y(I)*(C3+Y(I)*(C2+Y(I)*(C1+Y(I)*C0)))))
     BB(I)=BD(I)/E(I)
     CC(I)=CD(I)/E(I)
     AAY(I)=AA(I)*Y(I)/TOU
     BBY(I)=BB(I)*Y(I)/TOU
 50  CONTINUE
     PRINT  3500
3500 FORMAT(// 7X,*AA(1),AA(2)=*,17X,*AA(3)=*,11X,*AA(4)=*,11X,
    1*AA(5)=*,11X,*AA(6)=*)
     PRINT  3550,RA,AIA,(AA(I),I=3,6)
3550 FORMAT(/12X,F8.5,*+/-I*,F8.5,9X,4(5X,F8.5,5X))
```

```
      PRINT  3600
3600  FORMAT(//17X,*BB(1),BB(2)=*,17X,*BB(3)=*,11X,*BB(4)=*,11X,
     1*BB(5)=*,11X,*BB(6)=*)
      PRINT 1550,  RB,BIB,(BB(I),I=3,6)
      PRINT  3700
3700  FORMAT(//17X,*CC(1),CC(2)=*,17X,*CC(3)=*,11X,*CC(4)=*,11X,
     1*CC(5)=*)
      PRINT 1550, RC,CIC,(CC(I),I=3,5)
      PRINT  38
38    FORMAT  (//13X,*AAY(3)=*,11X,*AAY(4)=*,11X,*AAY(5)=*)
      PRINT  3850,(AAY(I),I=3,5)
3850  FORMAT  (/5X,3(5X,F6.3,5X))
      PRINT  3875
3875  FORMAT  (//13X,*BBY(3)=*,11X,*BBY(4)=*,11X,*BBY(5)=*)
      PRINT 3850 , (BBY(I),I=3,5)
      PRINT  3017
      PRINT  3  1
      PRINT 3020
      PRINT  3  1
      PRINT  30 7
      PRINT 190,AKA,Y(3),Y(5),WA,(AA(I),Y(I),I=3,4),AA(5),AA(6)
190   FORMAT(//5X,*.........=*,F10.6,*EXP(SIGMA*,F10.6,*)COS(SIGMA*,
     1F10.6,  2H+,F10.6  ,2H)+/5X,2(F10.6,*EXP(SIGMA*,F10.6,2H)+),
     2F10.6,*SIGMA+*,F10.6)
200   FORMAT(//5X,*.........=*,F10.6,*EXP(SIGMA*,F10.6,*)COS(SIGMA*,
     1F10.6,  2H+,F10.6  ,2H)+/5X,2(F10.6,*EXP(SIGMA*,F10.6,2H)+),
     2F10.6)
210   FORMAT(//5X,*.........=*,F10.6,*EXP(SIGMA*,F10.6,*)COS(SIGMA*,
     1F10.6,2(2H+,F10.6),2H)+/5X,2(F10.6,*EXP(SIGMA*,F10.6,2H)+),
     2F10.6)
      PRINT190,BKB,Y(3),Y(5),WB,(BB(I),Y(I),I=3,4),BB(5),BB(6)
      PRINT  200,CKC,Y(3),Y(5),WC,(CC(I),Y(I),I=3,4),CC(5)
      PRINT  21 ,AKAA,Y(3),Y(5),WA,WAA,(AAY(I),Y(I),I=3,4),AAY(5)
      PRINT  21 ,AKAB,Y(3),Y(5),WB,WAA,(BBY(I),Y(I),I=3,4),BBY(5)
1045  T=0.0
      DO 1040 LICK=1,120
      SIGMA=T/TAU
      SUM =0.
      SUMP=0.0
      SMP =0.0
      SMUP =0.0
      TOT=0.0
      DO  1050  I=3,4
      P=SIGMA*Y(I)
      ZD(I)=EXP(P)
      BA(I)=AA(I)*ZD(I)
      BAAY(I)=AAY(I)*ZD(I)
      AB(I)=BB(I)*ZD(I)
      ABBY(I)=BBY(I)*ZD(I)
      BC(I)=CC(I)*ZD(I)
      SUM=SUM+BA(I)
      SUMP=SUMP+BAAY(I)
      SMP=SMP+AB(I)
      SMUP=SMUP+ABBY(I)
      TOT=TOT+BC(I)
```

```
1950 CONTINUE
     COSA=COS(SIGMA*Y(5)+WA)
     COSB=COS(SIGMA*Y(5)+WB)
     COSC=COS(SIGMA*Y(5)+WC)
     COSP=COS(SIGMA*Y(5)+WA+WAA)
     COSR=COS(SIGMA*Y(5)+WB+WAA)
     EXPR=EXP(SIGMA*Y(1))
     PHI=AKA*EXPR*COSA+SUM+AA(5)*SIGMA+AA(6)
     PSI=BKB*EXPR*COSB+SMP+BB(5)*SIGMA+BB(6)
     BETA=CKC*EXPR*COSC+TOT+CC(5)
     ROLVEL=AKAA*EXPR*COSP+SUMP+AAY(5)
     YAWVEL=AKBB*EXPR*COSR+SMUP+BBY(5)
     PHID(LICK)=PHI*180./PI
     PSID(LICK)=PSI*180./PI
     BETAD(LICK)=BETA*180./PI
     ROVELD(LICK)=ROLVEL*180./PI
     YAVELD(LICK)=YAWVEL*180./PI
     TI(LICK)=T
     T=T+0. 5
104  CONTINUE
     GO TO 65
 115 DO 105 I=1,2
     AD(I)=A5+Y(I)*(A4+Y(I) *(A3+Y(I)*(A2+Y(I)*(A1+Y(I)*A0))))
     E(I)=Y(I)*(2.*A(6)+Y(I)*(3.*A(5)+Y(I)*(4.*A(4)+Y(I)*(5.*A(3)+
    1Y(I)*6.*A(2)))))
     AA(I)=AD(I)/E(I)
     BD(I)=B5+Y(I)*(B4+Y(I)*(B3+Y(I)*(B2+Y(I)*(B1+Y(I)*B0))))
     BB(I)=BD(I)/E(I)
     CD(I)=Y(I)*(C4+Y(I)*(C3+Y(I)*(C2+Y(I)*(C1+Y(I)*C0))))
     CC(I)=CD(I)/E(I)
     AAY(I)=AA(I)*Y(I)/TOU
     BBY(I)=BB(I)*Y(I)/TOU
105  CONTINUE
125  R1=Y(3)
     R2=Y(3)**2-Y(7)**2
     R3=Y(3)**3-3.*Y(3)*Y(7)**2
     R4=Y(3)**4-6.*(Y(3)*Y(7))**2+Y(7)**4
     R5=Y(3)**5-10.*Y(3)**3*Y(7)**2+5.*Y(3)*Y(7)**4
     AI1=Y(7)
     AI2=2.*Y(7)*Y(3)
     AI3=3.*Y(3)**2*Y(7)-Y(7)**3
     AI4=4.*(Y(3)**3*Y(7)-Y(3)*Y(7)**3)
     AI5=(5.*Y(3)**4-10.*(Y(3)*Y(7))**2+Y(7)**4)*Y(7)
     AX1=A0*R5+A1*R4+A2*R3+A3*R2+A4*R1+A5
     AY1=A0*AI5+A1*AI4+A2*AI3+A3*AI2+A4*AI1
     X2=6.*A(2)* R5+5.*A(3)* R4+4.*A(4)* R3+3.*A(5)*  R2+2.*A(6)*  R1
     Y2=6.*A(2)*AI5+5.*A(3)*AI4+4.*A(4)*AI3+3.*A(5)*AI2+2.*A(6)*AI1
     Z=X2**2+Y2**2
     RAD=(AX1*X2+AY1*Y2)/Z
     AIAD=(X2*AY1-AX1*Y2)/Z
     BX1=B0*R5+B1*R4+B2*R3+B3*R2+B4*R1+B5
     BY1=B0*AI5+B1*AI4+B2*AI3+B3*AI2+B4*AI1
     RBD=(BX1*X2+BY1*Y2)/Z
     BIBD=(X2*BY1-BX1*Y2)/Z
     CX1=C0*R5+C1*R4+C2*R3+C3*R2+C4*R1
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```
      CY1=C0*AI5+C1*AI4+C2*AI3+C3*AI2+C4*AI1
      RCD=(CX1*X2+CY1*Y2)/Z
      CICD=(X2*CY1-CX1*Y2)/Z
      AKAD=2.*SQRT(RAD**2+AIAD**2)
      BKBD=2.*SQRT(RBD**2+BIBD**2)
      CKCD=2.*SQRT(RCD**2+CICD**2)
      CALL  ARCTAN(RAD,AIAD,WAD)
      CALL ARCTAN(RBD,BIBD,WBD)
      CALL ARCTAN(RCD,CICD,WCD)
      YAKAD=SQRT(Y(3)**2+Y(7)**2)/TOU
      CALL ARCTAN(Y(3),Y(7),WAAD)
      AKAAD=AKAD*YAKAD
      AKABD=BKBD*YAKAD
      IF(Y(5).EQ.0.0)  GO TO 655
    6 PRINT 4400
 4400 FORMAT(//17X,*AA(1),AA(2)=*,17X,*AA(3),AA(4)=*,17X,*AA(5)=*,11X,
     1 *AA(6)=*)
      PRINT 4450 ,RA,AIA,RAD,AIAD,AA(5),AA(6)
 4450 FORMAT(/5X,2(5X,F8.5,*+/-I*,F8.5,5X),10X,2(F8.5,5X))
      PRINT 4500
 4500 FORMAT(//17X,*BB(1),BB(2)=*,17X,*BB(3),BB(4)=*,17X,*BB(5)=*,11X,
     1*BB(6)=*)
      PRINT 4450,RB,BIB,RBD,BIBD,BB(5),BB(6)
      PRINT  4600
 4600 FORMAT(//17X,*CC(1),CC(2)=*,17X,*CC(3),CC(4)=*,17X,*CC(5)=*)
      PRINT 4450,RC,CIC,RCD,CICD,CC(5)
      PRINT  4700
 4700 FORMAT(//19X,*AAY(5)=*,11X,*BBY(5)=*)
      PRINT  4750,AAY(5),BBY(5)
  475 FORMAT(5X,2(5X,F8.5,5X))
      PRINT  3007
      PRINT 30
      PRINT 302
      PRINT  3001
      PRINT  3017
      PRINT 27 ,AKA,Y(1),Y(5),WA,AKAD,Y(3),Y(7),WAD,AA(5),AA(6)
      PRINT 27 ,BKB,Y(1),Y(5),WB,BKBD,Y(3),Y(7),WBD,BB(5),BB(6)
      PRINT29C, CKC,Y(1),Y(5),WC,CKCD,Y(3),Y(7),WCD,BB(5),BB(6)
      PRINT 3C0,AKAA,Y(1),Y(5),WA,WAA,AKAAD,Y(3),Y(7),WCD,CC(5)
      PRINT 30 , AKAB,Y(1),Y(5),WB,WAA,AKABD,Y(3),Y(7),WAD,WAAD,AAY(5)
  270 FORMAT(//5X,*.........=*,2(5X,F10.6,*EXP(SIGMA*,F10.6,*)CCS(
     1SIGMA*, F10.6,  1H+,F10.6 ,2H)+/),F10.6,*SIGMA+*,F10.6)
  290 FORMAT(//5X,*.........=*,2(5X,F10.6,*EXP(SIGMA*,F10.6,*)CCS(
     1SIGMA*, F1 .6,  1H+,F10.6 ,2H)+/),F10.6)
  300 FORMAT(//5X,*.........=*,2(5X,F10.6,*EXP(SIGMA*,F10.6,*)CCS(
     1SIGMA*,F10.6,2(1H+,F10.6),2H)+/),F10.6)
 1065 T=0.0
      DO 106 KICK=1,120
      SIGMA=T/TOU
      COSA=COS(SIGMA*Y(5)+WA)
      COSB=COS(SIGMA*Y(5)+WB)
      COSC=COS(SIGMA*Y(5)+WC)
      COSP=COS(SIGMA*Y(5)+WA+WAA)
      COSR=COS(SIGMA*Y(5)+WB+WAA)
      EXPR=EXP(SIGMA*Y(1))
```

```
      COSAD=CCS(SIGMA*Y(7)+WAD)
      COSBD=CCS(SIGMA*Y(7)+WBD)
      COSCD=CCS(SIGMA*Y(7)+WCD)
      COSPD=CCS(SIGMA*Y(7)+WAD+WAAD)
      COSRD=CCS(SIGMA*Y(7)+WBD+WAAD)
      EXPRD=EXP(SIGMA*Y(3))
      PHI=AKA*EXPR*COSA+AA(5)*SIGMA+AA(6)+AKAD*EXPRD*COSAD
      PSI=BKB*EXPR*COSB+BB(5)*SIGMA+BB(6)+BKBD*EXPRD*COSBD
      BETA=CKC*EXPR*COSC+CC(5)+CKCD*EXPRD*COSCD
      ROLVEL=AKAA*EXPR*COSP+AAY(5)+AKAAD*EXPRD*COSPD
      YAWVEL=AKAB*EXPR*COSR+BBY(5)+AKABD*EXPRD*COSRD
      PHID(KICK)=PHI*180./PI
      PSID(KICK)=PSI*180./PI
      BETAD(KICK)=BETA*180./PI
      ROVELD(KICK)=ROLVEL*180./PI
      YAVELD(KICK)=YAWVEL*180./PI
      TI(KICK)=T
      T=T+0.05
 306  CONTINUE
      GO TO 65
 650  PRINT 3900
 3900 FORMAT(//10X,*AA(1)=*,11X,*AA(2)=*,17X,*AA(3),AA(4)=*,17X,
     1*AA(5)=*,11X,*AA(6)=*)
      PRINT  3950,AA(1),AA(2),RAD,AIAD,AA(5),AA(6)
 3950 FORMAT(/5X,2(5X,F8.5,5X),F8.5,*+-I*,F8.5,10X,2(5X,F8.5,5X))
      PRINT  4000
 4000 FORMAT(//10X,*BB(1)=*,11X,*BB(2)=*,17X,*BB(3),BB(4)=*,17X,
     1*BB(5)=*,11X,*BB(6)=*)
      PRINT  3950,BB(1),BB(2),RBD,BIBD,BB(5),BB(6)
      PRINT  4100
 4100 FORMAT(//10X,*CC(1)=*,11X,*CC(2)=*,17X,*CC(3),CC(4)=*,17X,
     1*CC(5)=*)
      PRINT  3950,CC(1),CC(2),RCD,CICD,CC(5)
      PRINT  4200
 4200 FORMAT(//10X,*AAY(1)=*,11X,*AAY(2)=*,11X,*AAY(5)=*)
      PRINT  3850,AAY(1),AAY(2),AAY(5)
      PRINT  4300
 4300 FORMAT(//10X,*BBY(1)=*,11X,*BBY(2)=*,11X,*BBY(5)=*)
      PRINT  3850,BBY(1),BBY(2),BBY(5)
      PRINT  3017
      PRINT5001
      PRINT  3023
      PRINT  3001
      PRINT 3027
      PRINT  380,(AA(I),Y(I),I=1,2),AKAD,Y(3),Y(7),WAD,AA(5),AA(6)
      PRINT  380,(BB(I),Y(I),I=1,2),BKBD,Y(3),Y(7),WBD,BB(5),BB(6)
      PRINT  390,(CC(I),Y(I),I=1,2),CKCD,Y(3),Y(7),WCD,CC(5)
      PRINT  400,(AAY(I),Y(I),I=1,2),AKAAD,Y(3),Y(7),WAD,WAAD,AAY(5)
      PRINT  400,(BBY(I),Y(I),I=1,2),AKABD,Y(3),Y(7),WBD,WAAD,BBY(5)
 380  FORMAT(//5X,*.........=*,2(F10.6,*EXP(SIGMA*,F10.6,*)+*)/
     15X,F10.6,*EXP(SIGMA*,F10.6,*)COS(SIGMA*,F10.6,   1H+,F10.6 ,
     2*)+*,F10.6,*SIGMA+*,F10.6)
 390  FORMAT(//5X,*.........=*,2(F10.6,*EXP(SIGMA*,F10.6,*)+*)/
     15X,F10.6,*EXP(SIGMA*,F10.6,*)COS(SIGMA*,F10.6,   1H+,F10.6 ,
     2*)+*,F10.6)
```

```
 400   FORMAT(//5X,*.........=*,2(F10.6,*EXP(SIGMA*,F10.6,*)+*)/
      15X,F10.6,*EXP(SIGMA*,F10.6,*)COS(SIGMA*,F10.6,2(1H+,F10.6),
      2*)+*,F10.3)
 1025  T=0.0
       DO 1020 KILL=1,120
       SIGMA=T/TOU
        SUM=0.0
       SUMP=0.0
       SMP =0.0
       SMUP =0.0
       TOT=0.0
       DO  1030 I=1,2
       P=SIGMA*Y(I)
       ZD(I)=EXP(P)
       BA(I)=AA(I)*ZD(I)
       BAAY(I)=AAY(I)*ZD(I)
       AB(I)=BB(I)*ZD(I)
       ABBY(I)=BBY(I)*ZD(I)
       BC(I)=CC(I)*ZD(I)
       SUM=SUM+BA(I)
       SUMP=SUMP+BAAY(I)
       SMP=SMP+AB(I)
       SMUP=SMUP+ABBY(I)
       TOT=TOT+BC(I)
 1030  CONTINUE
       COSAD=COS(SIGMA*Y(7)+WAD)
       COSBD=COS(SIGMA*Y(7)+WBD)
       COSCD=COS(SIGMA*Y(7)+WCD)
       COSPD=COS(SIGMA*Y(7)+WAD+WAAD)
       COSRD=COS(SIGMA*Y(7)+WBD+WAAD)
       EXPRD=EXP(SIGMA*Y(3))
       PHI=AKAD*EXPRD*COSAD+SUM+AA(5)*SIGMA+AA(6)
       PSI=BKBD*EXPRD*COSBD+SMP+BB(5)*SIGMA+BB(6)
       BETA=CKCD*EXPRD*COSCD+TOT+CC(5)
       ROLVEL=AKAAD*EXPRD*COSPD+SUMP+AAY(5)
       YAWVEL=AKABD*EXPRD*COSRD+SMUP+BBY(5)
       PHID(KILL)=PHI*180./PI
       PSID(KILL)=PSI*180./PI
       BETAD(KILL)=BETA*180./PI
       ROVELD(KILL)=ROLVEL*180./PI
       YAVELD(KILL)=YAWVEL*180./PI
       T1(KILL)=T
       T=T+0.05
 1020  CONTINUE
 65    CONTINUE
       PRINT 1900
       PRINT 3000
       PRINT 2365,(T1(J),PHID(J),PSID(J),BETAD(J),ROVELD(J),YAVELD(J),
      1J=1,120)
       PRINT 1900
       DO 52 I=1,120
       DO 51 J=1,3
       NP(J)=120
 51    TIME(J,I)=T1(I)
       PARAM(1,I)=PHID(I)
```

```
      PARAM(2,I)=PSID(I)
      PARAM(3,I)=BETAD(I)
      PARAM(4,I)=ROVELD(I)
      PARAM(5,I)=YAVELD(I)
52    CONTINUE
      CALL GRAPHM(TIME,PARAM,5,NP)
      PRINT 3027
      PRINT 3001
      PRINT 3011 ,BTDO
      PRINT 3001
      PRINT 3027
      CALL GRAF(T1,BETAD ,120,120,0,12,9)
      PRINT  3027
      PRINT 3001
      PRINT  3012
      PRINT 3010,BTDO
      PRINT  3001
      PRINT 3027
      CALL GRAF(T1,PHID  ,120,120,0,12,9)
      PRINT 3027
      PRINT 3001
      PRINT 3010
      PRINT 3010,BTDO
      PRINT 3001
      PRINT 3027
      CALL GRAF(T1,PSID  ,120,120,0,12,9)
      PRINT 3027
      PRINT 3001
      PRINT 3011
      PRINT 3011 ,BTDO
      PRINT 3001
      PRINT 3027
      CALL GRAF(T1,ROVELD,120,120,0,12,9)
      PRINT 3027
      PRINT 3001
      PRINT 3013
      PRINT 3011 ,BTDO
      PRINT 3001
      PRINT 3027
      CALL GRAF(T1,YAVELD,120,120,0,12,9)
      PRINT 3027
      PRINT 3001
      PRINT 3014
      PRINT 3011 ,BTDO
      PRINT 3001
      PRINT 3027
165   CONTINUE
2101  FORMAT (5F13.6)
2301  FORMAT (5F7.4)
2501  FORMAT(3F5.5)
1900  FORMAT(1X,130(1H*)//)
49    FORMAT(//16X,*A=*,F11.5,5X,*B=*,F11.5,5X,*C=*,F11.5,5X,
     1*D=*,F11.5,5X,*E=*,F11.5//)
2365  FORMAT(16XF7.4,7XF8.4,5XF8.4,7XF8.4,11XF8.4,16XF8.4)
3001  FORMAT(39X,1H*,52X,1H*)
```

```
3002 FORMAT(//60X*CESSNA 182*/60X10(1H-)//20X*MASS AND GEOMETRICAL CHAR
    1ACTERISTICS*/20X36(1H-)//30X*WEIGHT*10X*MOMENT OF INERTIA*10X*SPAN
    2*10X*WING AREA*/30X*(LBS)*14X*(SLUGS SQ.FT)*11X*(FT)*11X*(SQ.FT)*/
    346X*IXX*1 X*IZZ*//30XF7.1,F14.1,F13.1,F15.2,F16.2)
3003 FORMAT(//20X*FLIGHT CONDITIONS*/20X17(1H-)//30X*VELOCITY*10X*DENSI
    1TY*10X*  LIFT  *10X* ANGLE OF *10X*FLIGHT PATH*/32X*(MPH)* 8X*(SLU
    2GS/CU.FT)*6X*COEFFICIENT*7X*ATTACK(DEG)*10X*ANGLE(DEG)*/32X,F6.1,F
    320.6,F17.3,F16.2,F18.2)
3004 FORMAT(//20X*STABILITY DERIVATIVES*/21X,20(1H-)//32X*SIDESLIP*12X*
    1ROLL RATE*12X*YAW RATE*13X*CONTROL*/30X*CYB=*F7.4,11X*CYP=*F7.4,10
    2X*CYR=*F7.4,10X*CYC=*F7.4/30X*CLB=*F7.4,11X*CLP=*F7.4,10X*CLR=*F7.
    34,10X*CLC=*F7.4/3 X*CNB=*F7.4,11X*CNP=*F7.4,10X*CNR=*F7.4,10X*CNC=
    4*F7.4)
3007 FORMAT(//20X*INITIAL CONDITIONS*/20X18(1H-)//30X*ANGLES(DEG)*10X*R
    1ATES(DEG/SEC)*/30X*ROLL*F8.1,10X*ROLL*F6.1/30X*YAW*F9.1,10X*YAW*F7
    2.1/29X*SIDESLIP*F5.1)
3008 FORMAT(// 0X*VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL*/
    140X51(1H-)/54X*AND YAW RATES WITH TIME*/54X23(1H-)/18X*TIME*10X*PH
    2I*10X*PSI*10X*BETA*10X*ROLL VELOCITY*10X*YAW VELOCITY*/18X*(SEC)*8
    3X*(DEG)*8X*(DEG)*8X*(DEG)*12X*(DEG/SEC)*14X*(DEG/SEC)*)
3005 FORMAT(1H )
3006 FORMAT(1H )
3009 FORMAT(39X,1H*,17X,*ROLL ANGLE VS TIME*,17X,1H*)
3010 FORMAT(39 ,1H*,9X,*DISTURBANCE-- BETA=*,F6.2,*DEGREES*,11X,1H*)
3011 FORMAT(39X,1H*,17X,*YAW ANGLE VS TIME *,17X,1H*)
3012 FORMAT(39 ,1H*,15X,*SIDESLIP ANGLE VS TIME*,15X,1H*)
3013 FORMAT(39X,1H*,14X,*ROLLING VELOCITY VS TIME*,14X,1H*)
3014 FORMAT(39X,1H*,14X,*YAWING VELOCIOY VS TIME *,14X,1H*)
3022 FORMAT(39X,1H*,6X*COEFFICIENTS OF CHARACTERISTIC EQUATION *6X1H*)
3023 FORMAT(39X,1H*,16X,*EQUATIONS OF MOTION *,16X,1H*)
3027 FORMAT(39X,54(1H*))
     STOP
     END
C
     SUBROUTINE  QUART (A,Y)
C    THIS PROGRAMME CALLED SUBROUTINE QUART SOLVES QUARTIC EQUATIONS
C    BY CALLING THE CUBIC AND QUADRATIC SUBROUTINES
     DIMENSION  A(6),Y(8), AC(4), AQ(3), BQ(3),RT(3)
     EQUIVALENCE(AQ,BQ)
     C3=A(3)/A(2)
     C2=A(4)/A(2)
     C1=A(5)/A(2)
     C0=A(6)/A(2)
     C=C3/2.
     AC(1)=1.
     AC(2)=-C2
     AC(3)=C1*C3-4.*C0
     AC(4)=C0*(4.*C2-C3**2)-C1**2
     CALL CUBIC(AC,RT,RTI)
     IF(RTI)  2 ,10,2
10   IF(RT(1)-RT(2))  11,12,12
11   RT(1)=RT(2)
12   IF(RT(1)-RT(3))   13,20,20
13   RT(1)=RT(3)
20   B=RT(1)/2.
```

```
      IF(B**2-C2)  22,22,24
 22   D=0.
      CA=SQRT(C*C+2.*B-C2)
      GO TO 25
 24   D=SQRT(B*B-C2)
      CA=-(C1/2.-C*B)/D
 25   AQ(1)=1.
      AQ(2)=C-CA
      AQ(3)=B-D
      CALL QUAD(AQ,Y(1),Y(2),Y(5))
      BQ(2)=C+CA
      BQ(3)=B+D
      CALL QUAD(BQ,Y(3),Y(4),Y(7))
      Y(6)=-Y(5)
      Y(8)=-Y(7)
      PRINT  65,Y(1),Y(5),Y(2),Y(6)
      PRINT  65,Y(3),Y(7),Y(4),Y(8)
  65  FORMAT(//10X,*THE ROOTS ARE*,E15.8,2H+I,E15.8//24X,E15.8,2H+I,
     1E15.8)
      RETURN
      END
C
      SUBROUTINE       CUBIC (A,XR,XI)
C      THIS SUBROUTINE SOLVES CUBIC EQUATIONS AND IS CALLED BY THE SUB Q
C     ROUTINE QUARTIC AND IN ITS TURN CALLS SUBROUTINE QUAD.
      DIMENSION  A(4),XR(3),AQ(3)
      IPATH=2
      EX=1.0/3.
      IF(A(4))  1006,1004,1006
1004  XR(1)=0.0
      GO TO 1034
1006  A2=A(1)**2
      Q=(27.*A2*A(4)-9.*A(1)*A(2)*A(3)+2.*A(2)**3)/(54.*A2*A(1))
      IF(Q)   1010,1008,1014
1008  Z=0.0
      GO TO 1032
1010  Q=-Q
      IPATH=1
1014  P=(3.*A(1)*A(3)-A(2)**2)/(9.*A2)
      ARG=P**3+Q*Q
      IF(ARG)   1016,1018,1020
1016  Z=-2.*SQRT(-P)*COS(ATAN(SQRT(-ARG)/Q)/3.)
      GO TO 1028
1018  Z=-2.*Q**EX
      GO TO 1028
1020  SARG=SQRT(ARG)
      IF(P)   1022,1024,1026
1022  Z=-(Q+SARG)**EX-(Q-SARG)**EX
      GO TO 1028
1024  Z=-(2.*Q)**EX
      GO TO 1028
1026  Z=(SARG-Q)**EX-(SARG+Q)**EX
1028  GO TO (1030,1032),IPATH
1030  Z=-Z
1032  XR(1)=(3.*A(1)*Z-A(2))/(3.*A(1) )
```

```
1034  AQ(1)=A(1)
      AQ(2)=A(2)+XR(1)*A(1)
      AQ(3)=A(3)+XR(1)*AQ(2)
      CALL QUAD(AQ,XR(2),XR(3),XI)
      RETURN
      END
C
      SUBROUTINE  QUAD(A,XR1,XR2,XI)
C      THIS SUBROUTINE SOLVES QUADRATIC EQUATIONS AND IS CALLED BY BOTH
C     THE QUARTIC AND CUBIC SUBROUTINES
      DIMENSION   A(3)
      X1=-A(2)/(2.*A(1))
      DISC=X1**2-A(3)/A(1)
      IF (DISC)  10,20,20
10    X2=SQRT(-DISC)
      XR1=X1
      XR2=X1
      XI=X2
      GO TO 30
20    X2=SQRT(DISC)
      XR1=X1+X2
      XR2=X1-X2
      XI=0.0
30    RETURN
      END
C
      SUBROUTINE  ARCTAN (X,Y,Z)
C  PROGRAMME TO EVALUATE ARCTAN Z AND TO DETERMINE THE PRUPR QUADRANT
      PI=4.*ATAN(1.)
      Z=  ATAN  (ABS(Y/X))
      ZD=Z*180./PI
      IF (Y)  20,20,30
 20   IF  (X)  15,25,35
 25   Z=Z+PI
      ZD=ZD+180.
      GO TO 80
 35   Z=2.*PI-Z
      ZD=360.-ZD
      GO TO 80
 30   IF  (X) 40,40,80
 40   Z=PI-Z
      ZD=180.-ZD
 80   PRINT  45,Z,ZD
45    FORMAT(5X,2(F9.4,5X))
      RETURN
      END
C **** THE PLCT SUBROUTINES 'GRAF' AND GRAPHM ARE GIVEN IN APPENDIX 'E'.
```

APPENDIX 'D'

FLOW CHART OF SOLUTION OF LATERAL EQUATIONS OF MOTION BY FOURTH ORDER RUNGE-KUTTA METHOD

FLOW CHART FOR SOLUTION OF LATERAL EQUATIONS OF MOTION BY FOURTH ORDER RUNGE-KUTTA METHOD

START

↓

INPUT. AIRPLANE CHARACTERISTICS & FLIGHT CONDITIONS

$m, I_{xx}, I_{zz}, \eta, \rho, S, b, V, C_L, \gamma, \epsilon, \alpha$

↓

INPUT: STABILITY DERIVATIVES.

$C_{l_\beta}, C_{n_\beta}, C_{y_\beta}, C_{l_p}, C_{n_p}, C_{y_p}, C_{l_r}, C_{n_r}, C_{y_r}, C_{l_\zeta}, C_{n_\zeta}, C_{y_\zeta}$

↓

INPUT: INITIAL CONDITIONS.

$\phi_0, (D\phi)_0, \psi_0, (D\psi)_0, \beta_0$

↓

NON-DIMENSIONALISED PARAMETERS

$$K_{x_0} = \frac{k_{x_0}}{b}$$

$$K_{z_0} = \frac{k_{z_0}}{b}$$

$$K_x^2 = K_{x_0}^2 \cos^2\eta + K_{z_0}^2 \sin^2\eta$$

$$K_z^2 = K_{z_0}^2 \cos^2\eta + K_{x_0}^2 \sin^2\eta$$

$$K_{xz} = (K_{z_0}^2 - K_{x_0}^2) \sin\eta \cos\eta$$

$$K_1 = \frac{K_{xz}}{K_x^2}$$

$$K_2 = \frac{K_{xz}}{K_z^2}$$

↓

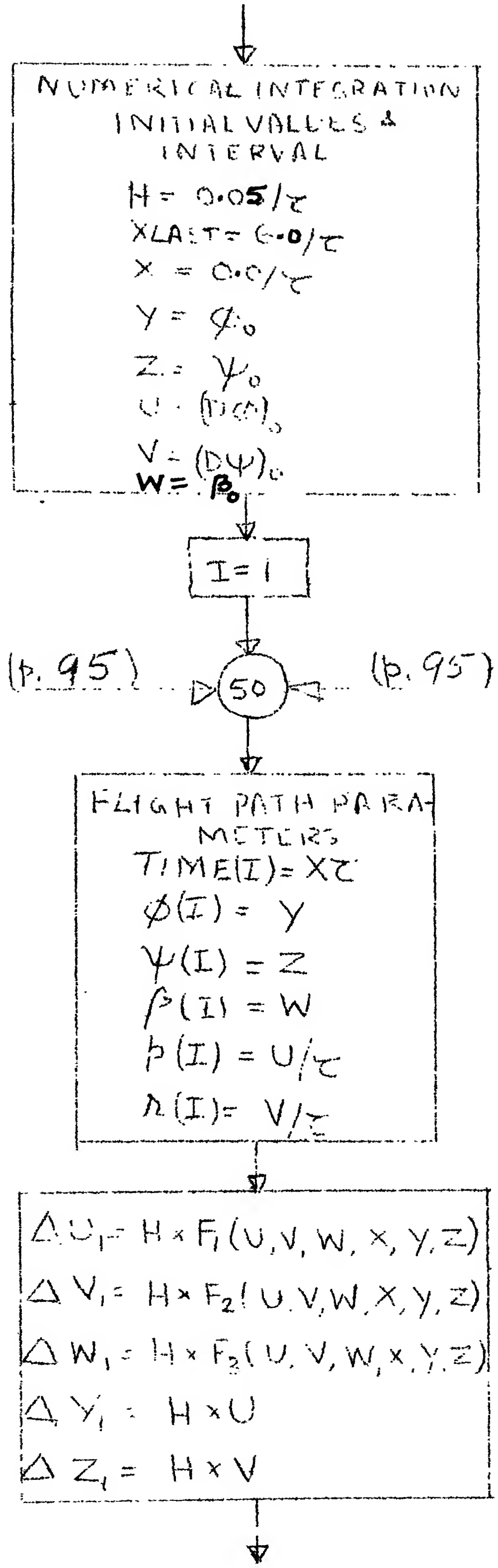
$$\tau = m/\rho S V$$
$$\sigma = t/\tau$$
$$\mu = m/\rho S b$$

$$l_\beta = \frac{\mu}{2K_x^2} C_{l_\beta}$$
$$n_\beta = \frac{\mu}{2K_z^2} C_{n_\beta}$$
$$y_\beta = \frac{1}{2} C_{y_\beta}$$
$$l_p = \frac{1}{4K_x^2} C_{l_p}$$
$$n_p = \frac{1}{4K_z^2} C_{n_p}$$
$$y_p = \frac{1}{4\mu} C_{y_p}$$
$$l_r = \frac{1}{4K_x^2} C_{l_r}$$
$$n_r = \frac{1}{4K_z^2} C_{n_r}$$
$$y_r = \frac{1}{4\mu} C_{y_r}$$
$$l_c = \frac{\mu}{2K_x^2} C_{l_c}$$
$$n_c = \frac{\mu}{2K_z^2} C_{n_c}$$
$$y_c = \frac{1}{2} C_{y_c}$$

DEFINE THE FUNCTIONS & COEFFICIENTS

$$DEN = 1 - K_1 K_2$$
$$C_1 = (l_p - K_1 n_p)/DEN$$
$$C_2 = (l_r - K_1 n_r)/DEN$$
$$C_3 = (l_\beta - K_1 n_\beta)/DEN$$
$$C_4 = (l_c - K_1 n_c)/DEN$$
$$D_1 = (n_p - K_2 l_p)/DEN$$
$$D_2 = (n_r - K_2 l_r)/DEN$$
$$D_3 = (n_\beta - K_2 l_\beta)/DEN$$
$$D_4 = (n_c - K_2 l_c)/DEN$$
$$E_1 = y_p$$
$$E_2 = y_r - 1$$
$$E_3 = y_\beta$$
$$E_4 = y_c$$
$$E_5 = 0.5\, C_L$$
$$E_6 = 0.5\, C_L \tan \gamma$$

$$F_1(U, V, W, X, Y, Z) = C_1 U + C_2 V + C_3 W + C_4$$
$$F_2(U, V, W, X, Y, Z) = D_1 U + D_2 V + D_3 W + D_4$$
$$F_3(U, V, W, X, Y, Z) = E_1 U + E_2 V + E_3 W + E_4 + E_5 Y + E_6 Z$$

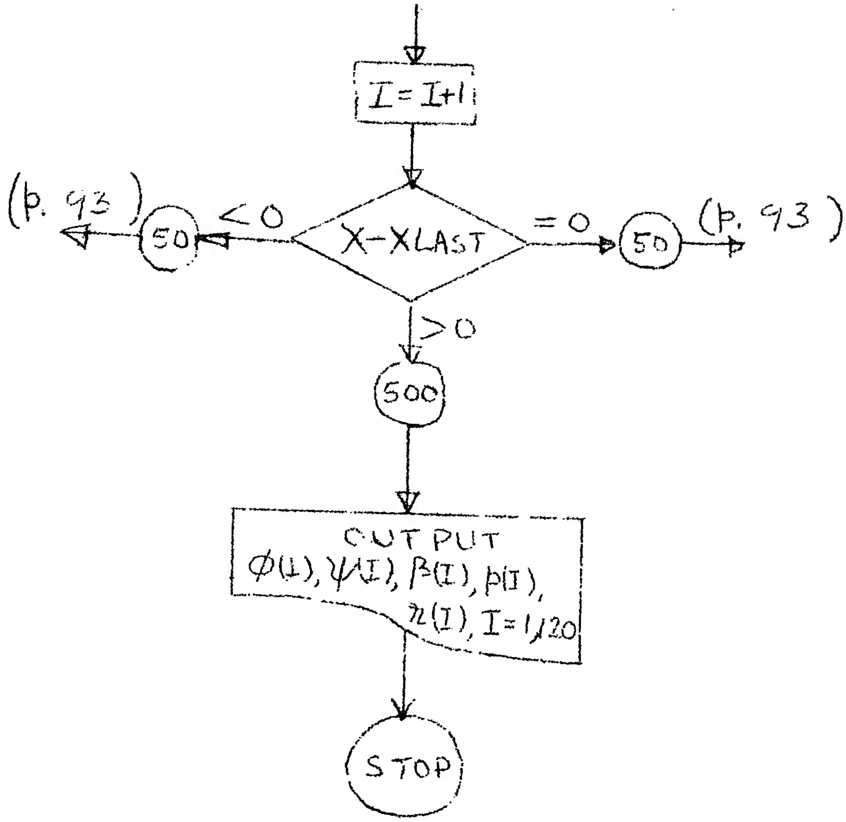

NUMERICAL INTEGRATION
INITIAL VALUES & INTERVAL
H = 0.05/τ
XLAST = 6.0/τ
X = 0.0/τ
Y = φ₀
Z = ψ₀
U = (Dφ)₀
V = (Dψ)₀
W = β₀
I = 1
(p. 95)
50
(p. 95)
FLIGHT PATH PARAMETERS
TIME(I) = Xτ
φ(I) = Y
ψ(I) = Z
β(I) = W
p(I) = U/τ
r(I) = V/τ
ΔU₁ = H × F₁(U, V, W, X, Y, Z)
ΔV₁ = H × F₂(U, V, W, X, Y, Z)
ΔW₁ = H × F₃(U, V, W, X, Y, Z)
ΔY₁ = H × U
ΔZ₁ = H × V

$$\Delta U_2 = H \times F_1(U+\Delta U_1, V+\Delta V_1, W+\Delta W_1, X+H/2, Y+\Delta Y_1, Z+\Delta Z_1)$$
$$\Delta V_2 = H \times F_2(U+\Delta U_1, V+\Delta V_1, W+\Delta W_1, X+H/2, Y+\Delta Y_1, Z+\Delta Z_1)$$
$$\Delta W_2 = H \times F_3(U+\Delta U_1, V+\Delta V_1, W+\Delta W_1, X+H/2, Y+\Delta Y_1, Z+\Delta Z_1)$$
$$\Delta Y_2 = H \times (U+\Delta U_1)$$
$$\Delta Z_2 = H \times (V+\Delta V_1)$$

$$\Delta U_3 = H \times F_1(U+\Delta U_2, V+\Delta V_2, W+\Delta W_2, X+H/2, Y+\Delta Y_2, Z+\Delta Z_2)$$
$$\Delta V_3 = H \times F_2(U+\Delta U_2, V+\Delta V_2, W+\Delta W_2, X+H/2, Y+\Delta Y_2, Z+\Delta Z_2)$$
$$\Delta W_3 = H \times F_3(U+\Delta U_2, V+\Delta V_2, W+\Delta W_2, X+H/2, Y+\Delta Y_2, Z+\Delta Z_2)$$
$$\Delta Y_3 = H \times (U+\Delta U_2)$$
$$\Delta Z_3 = H \times (V+\Delta V_2)$$

$$\Delta U_4 = H \times F_1(U+\Delta U_3, V+\Delta V_3, W+\Delta W_3, X+H, Y+\Delta Y_3, Z+\Delta Z_3)$$
$$\Delta V_4 = H \times F_2(U+\Delta U_3, V+\Delta V_3, W+\Delta W_3, X+H, Y+\Delta Y_3, Z+\Delta Z_3)$$
$$\Delta W_4 = H \times F_3(U+\Delta U_3, V+\Delta V_3, W+\Delta W_3, X+H, Y+\Delta Y_3, Z+\Delta Z_3)$$
$$\Delta Y_4 = H \times (U+\Delta U_3)$$
$$\Delta Z_4 = H \times (V+\Delta V_2)$$

$$X = X+H$$
$$Y = Y+\frac{1}{6}(\Delta Y_1+2\Delta Y_2+2\Delta Y_3+\Delta Y_4)$$
$$Z = Z+\frac{1}{6}(\Delta Z_1+2\Delta Z_2+2\Delta Z_3+\Delta Z_4)$$
$$U = U+\frac{1}{6}(\Delta U_1+2\Delta U_2+2\Delta U_3+\Delta U_4)$$
$$V = V+\frac{1}{6}(\Delta V_1+2\Delta V_2+2\Delta V_3+\Delta V_4)$$
$$W = W+\frac{1}{6}(\Delta W_1+2\Delta W_2+2\Delta W_3+\Delta W_4)$$



I = I+1
X−XLAST
< 0
50
(p. 93)
= 0
50
(p. 93)
> 0
500
OUTPUT
φ(I), ψ(I), β(I), p(I),
n(I), I=1,120
STOP

APPENDIX 'E'

DIGITAL COMPUTER PROGRAMME LISTING FOR SOLUTION OF LATERAL EQUATIONS OF MOTION BY FOURTH ORDER RUNGE-KUTTA METHOD

```
C
C **** THIS PROGRAMME SOLVES THE LATERAL EQUATIONS OF MOTION OF AN  AIR-
C **** PLANE BY FOURTH ORDER RUNGE-KUTTA METHOD AND PREDICTS THE TIME
C **** HISTORY OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL AND YAW RATES.
      COMMON/VAM1/C1,C2,C3 ,C4
      COMMON/VAM2/D1,D2,D3,D4
      COMMON/VAM3/E1,E2,E3,E4,E5,E6
      DIMENSICN TIME(5,125),PARAM(5,125),NP(5)
      DIMENSICNPHID(200),PSID(200),BETAD(200),ROVELD(200),YAVELD(200),
     1T1(200)
      COMMON/GPHM/ISCALE,NPLCT,NOPRNT,IHSIZE,JVSIZE
      NPLOT=1
      IHSIZE=12
      JVSIZE=6
      NOPRNT=1
      READ 2101,WEIT,XIKO,ZIKO,SPAN,AREA
      READ 2301,PHIO,DPHIO,PSIO,DPSIO,BTDO
 2301 FORMAT (5F7.4)
      DO 650 L=1,2
      READ 2101,VELM,RHO,EPSD,ALFAD,GAMMAD
 2101 FORMAT (5F13.6)
      DO 65 LJK=1,2
      READ 2501,CLB,CNB,CYB
      READ 2501,CLP,CNP,CYP
      READ 2501,CLR,CNR,CYR
      READ 2501,CLC,CNC,CYC
 2501 FORMAT(3F8.5)
 1900 FORMAT(1X,130(1H*)//)
      GRTY=32.2
      PI=4.*ATAN(1.)
      PID=180./PI
      AMASS=WEIT/GRTY
      ETA=(ALFAD-EPSD)/PID
      GAMMA=GAMMAD/PID
      BTAO=BTDO/PID
      VEL=VELM*88./60.
      CL=2.*WEIT/(RHO*VEL**2*AREA)
      CLG=.5*CL*TAN(GAMMA)
      XKO=SQRT(XIKO/AMASS)
      ZKO=SQRT(ZIKO/AMASS)
      XXK=((XKO*COS(ETA))**2+(ZKO*SIN(ETA))**2)/(SPAN**2)
      ZZK=((ZKO*COS(ETA))**2+(XKO*SIN(ETA))**2)/(SPAN**2)
      XZK=((ZKO**2-XKO**2)*SIN(ETA)*COS(ETA))/(SPAN**2)
      AK1=XZK/XXK
      AK2=XZK/ZZK
      TOU=AMASS/(RHO*AREA*VEL)
      AMU=AMASS/(RHO*AREA*SPAN)
      SLB=.5*AMU/XXK*CLB
      SLP=0.25*CLP/XXK
      SLR=0.25*CLR/XXK
      SLC=.5*AMU/XXK*CLC
      SNB=.5*AMU/ZZK*CNB
      SNP=0.25*CNP/ZZK
      SNR=0.25*CNR/ZZK
      SNC=.5*AMU/ZZK*CNC
```

```
      YB=.5 *CYB
      YP=.25*CYP/AMU
      YR=.25*CYR/AMU
      YC=.5*CYC
      DEN=1.-AK1*AK2
      C1=(SLP-AK1*SNP)/DEN
      C2=(SLR-AK1*SNR)/DEN
      C3=(SLB-AK1*SNB)/DEN
      C4=(SLC-AK1*SNC)/DEN
      D1=(SNP-AK2*SLP)/DEN
      D2=(SNR-AK2*SLR)/DEN
      D3=(SNB-AK2*SLB)/DEN
      D4=(SNC-AK2*SLC)/DEN
      E1=YP
      E2=YR-1.
      E4=YC
      E3=YB
      E5=0.5*CL
      E6=CLG
      H=0.05/TOU
      XLAST=6.0/TOU
      X=0.0/TOU
      U=DPHIO
      V=DPSIO
      W=BTAO
      Y=PHIO
      Z=PSIO
      I=1
      T1(I)=X*TOU
      PHID(I)=Y*PID
      PSID(I)=Z*PID
      BETAD(I)=W*PID
      ROVELD(I)=U*PID/TOU
      YAVELD(I)=V*PID/TOU
      DO51 J=1,5
      NP(J)=120
   51 TIME(J,I)=T1(I)
      PARAM(1,I)=PHID(I)
      PARAM(2,I)=PSID(I)
      PARAM(3,I)=BETAD(I)
      PARAM(4,I)=ROVELD(I)
      PARAM(5,I)=YAVELD(I)
      DELU1=H*F1(U,V,W,X,Y,Z)
      DELV1=H*F2(U,V,W,X,Y,Z)
      DELW1=H*F3(U,V,W,X,Y,Z)
      DELY1=H*U
      DELZ1=H*V
      DELU2=H*F1(U+DELU1/2.,V+DELV1/2.,W+DELW1/2.,X+H/2.,Y+DELY1/2.,
     Z+DELZ1/2.)
      DELV2=H*F2(U+DELU1/2.,V+DELV1/2.,W+DELW1/2.,X+H/2.,Y+DELY1/2.,
     Z+DELZ1/2.)
      DELW2=H*F3(U+DELU1/2.,V+DELV1/2.,W+DELW1/2.,X+H/2.,Y+DELY1/2.,
     Z+DELZ1/2.)
      DELY2=H*(U+DELU1/2.)
      DELZ2=H*(V+DELV1/2.)
```

```
      DELU3=H*F1(U+DELU2/2.,V+DELV2/2.,W+DELW2/2.,X+H/2.,Y+DELY2/2.,
     1Z+DELZ2/2.)
      DELV3=H*F2(U+DELU2/2.,V+DELV2/2.,W+DELW2/2.,X+H/2.,Y+DELY2/2.,
     1Z+DELZ2/2.)
      DELW3=H*F3(U+DELU2/2.,V+DELV2/2.,W+DELW2/2.,X+H/2.,Y+DELY2/2.,
     1Z+DELZ2/2.)
      DELY3=H*(U+DELU2/2.)
      DELZ3=H*(V+DELV2/2.)
      DELU4=H*F1(U+DELU3,V+DELV3,W+DELW3,X+H,Y+DELY3,Z+DELZ3)
      DELV4=H*F2(U+DELU3,V+DELV3,W+DELW3,X+H,Y+DELY3,Z+DELZ3)
      DELW4=H*F3(U+DELU3,V+DELV3,W+DELW3,X+H,Y+DELY3,Z+DELZ3)
      DELY4=H*(U+DELU3)
      DELZ4=H*(V+DELV3)
      X=X+H
      U=U+(DELU1+2.*DELU2+2.*DELU3+DELU4)/6.
      V=V+(DELV1+2.*DELV2+2.*DELV3+DELV4)/6.
      W=W+(DELW1+2.*DELW2+2.*DELW3+DELW4)/6.
      Y=Y+(DELY1+2.*DELY2+2.*DELY3+DELY4)/6.
      Z=Z+(DELZ1+2.*DELZ2+2.*DELZ3+DELZ4)/6.
      I=I+1
      IF(X-XLAST)  50,50,500
  500 CONTINUE
      PRINT 3005
      PRINT 3002,WEIT,XIKO,ZIKO,SPAN,AREA
      PRINT 3003,VELM,RHO,CL,ALFAD,GAMMAD
      PRINT 3004,CYB,CYP,CYR,CYC,CLB,CLP,CLR,CLC,CNB,CNP,CNR,CNC
      PRINT 3007,PHIO,DPHIO,PSIO,DPSIO,BTDO
      PRINT 3006
      PRINT 1900
      PRINT 3008
      PRINT 2365,(T1(J),PHID(J),PSID(J),BETAD(J),ROVELD(J),YAVELD(J),
     1J=1,120)
      PRINT 1900
      CALL GRAPHM(TIME,PARAM,5,NP)
      PRINT 3027
      PRINT 3001
      PRINT 3010,BTDO
      PRINT 3001
      PRINT 3027
      CALL GRAF(T1,BETAD ,120,120,0,12,9)
      PRINT 3027
      PRINT 3001
      PRINT 3012
      PRINT 3010,BTDO
      PRINT 3001
      PRINT 3027
      CALL GRAF(T1,PHID  ,120,120,0,12,9)
      PRINT 3027
      PRINT 3001
      PRINT 3009
      PRINT 3010,BTDO
      PRINT 3001
      PRINT 3027
      CALL GRAF(T1,PSID  ,120,120,0,12,9)
      PRINT 3027
```

```
      PRINT 3001
      PRINT 3011
      PRINT 3010,BTDD
      PRINT 3001
      PRINT 3027
      CALL GRAF(T1,ROVELD,120,120,0,12,9)
      PRINT 3027
      PRINT 3001
      PRINT 3013
      PRINT 3010,BTDD
      PRINT 3001
      PRINT 3027
      CALL GRAF(T1,YAVELD,120,120,0,12,9)
      PRINT 3027
      PRINT 3001
      PRINT 3014
      PRINT 3010,BTDD
      PRINT 3001
      PRINT 3027
   65 CONTINUE
  650 CONTINUE
 3001 FORMAT(39X,1H*,52X,1H*)
 3002 FORMAT(//60X*CESSNA 182*/60X10(1H-)//20X*MASS AND GEOMETRICAL CHAR
     1ACTERISTICS*/20X36(1H-)//30X*WEIGHT*10X*MOMENT OF INERTIA*10X*SPAN
     2*10X*WING AREA*/30X*(LBS)*14X*(SLUGS SQ.FT)*11X*(FT)*11X*(SQ.FT)*/
     346X*IXX*10X*IZZ*//30XF7.1,F14.1,F13.1,F15.2,F16.2)
 3003 FORMAT(//20X*FLIGHT CONDITIONS*/20X17(1H-)//30X*VELOCITY*10X*DENSI
     1TY*10X*  LIFT   *10X* ANGLE OF *10X*FLIGHT PATH*/32X*(MPH)* 8X*(SLU
     2GS/CU.FT)*6X*COEFFICIENT*7X*ATTACK(DEG)*10X*ANGLE(DEG)*/32X,F6.1,F
     320.6,F17.3,F16.2,F16.2)
 3004 FORMAT(//20X*STABILITY DERIVATIVES*/21X,20(1H-)//52X*SIDESLIP*12X*
     1ROLL RATE*12X*YAW RATE*13X*CONTROL*/30X*CYB=*F7.4,11X*CYP=*F7.4,10
     2X*CYR=*F7.4,10X*CYC=*F7.4/30X*CLB=*F7.4,11X*CLP=*F7.4,10X*CLR=*F7.
     34,10X*CLC=*F7.4/30X*CNB=*F7.4,11X*CNP=*F7.4,10X*CNR=*F7.4,10X*CNC=
     4*F7.4)
 3007 FORMAT(//20X*INITIAL CONDITIONS*/20X18(1H-)//30X*ANGLES(DEG)*10X*R
     1ATES(DEG/SEC)*/30X*ROLL*F8.1,10X*ROLL*F6.1/30X*YAW*F9.1,10X*YAW*F7
     2.1/29X*SIDESLIP*F5.1)
 3008 FORMAT(//40X*VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL*/
     140X51(1H-)/54X*AND YAW RATES WITH TIME*/54X23(1H-)/18X*TIME*10X*PH
     2I*10X*PSI*10X*BETA*10X*ROLL VELOCITY*10X*YAW VELOCITY*/19X*(SEC)*8
     3X*(DEG)*8X*(DEG)*8X*(DEG)*12X*(DEG/SEC)*14X*(DEG/SEC)*)
 2365 FORMAT(16XF7.4,7XF8.4,5XF8.4,7XF8.4,11XF8.4,16XF8.4)
 3005 FORMAT(1H1)
 3006 FORMAT(1H0)
 3009 FORMAT(39X,1H*,17X,*ROLL ANGLE VS TIME*,17X,1H*)
 3010 FORMAT(39X,1H*,9X,*DISTURBANCE-- BETA=*,F6.1,*DEGREE$*,11X,1H*)
 3011 FORMAT(39X,1H*,17X,*YAW ANGLE VS TIME *,17X,1H*)
 3012 FORMAT(39X,1H*,15X,*SIDESLIP ANGLE VS TIME*,15X,1H*)
 3013 FORMAT(39X,1H*,14X,*ROLLING VELOCITY VS TIME*,14X,1H*)
 3014 FORMAT(39X,1H*,14X,*YAWING VELOCIOY VS TIME *,14X,1H*)
 3027 FORMAT(39X,54(1H*))
      STOP
      END
```

```
      FUNCTION F1(U,V,W,X,Y,Z)
      COMMON/VAM1/C1,C2,C3 ,C4
      F1=C1*U+C2*V+C3*W+C4
      RETURN
      END
C
      FUNCTION F2(U,V,W,X,Y,Z)
      COMMON/ VAM2/D1,D2,D3,D4
      F2=D1*U+D2*V+D3*W+D4
      RETURN
      END
C
      FUNCTION F3(U,V,W,X,Y,Z)
      COMMON/ VAM3/E1,E2,E3,E4,E5,E6
      F3=E1*U+E2*V+E3*W+E4+E5*Y+E6*Z
      RETURN
      END
C
      SUBROUTINE GRAF(XD,YD,M,N,NPLOT,IHOR,JVER)
      DIMENSION XD(M),YD(M),PLOT(121,55)
      DATA BLANK,DOT,STAR,AI,DASH,OVER/1H ,1H.,1H*,1H ,1H ,1H+/
      IF(IHOR.GT.12.OR.JVER.GT.9)PRINT 1
      IF(IHOR.GT.12)IHOR=12
      IF(JVER.GT.9)JVER=9
      NXPLOT=0
      IF(NPLOT.EQ.1)  GO TO 20
      MINA=121
      LINA=55
      GO TO 30
   20 MINA=IHOR*10+1
      LINA=JVER*6+1
   30 NINA=MINA-1
      NEENA=MINA-10
      DO 40 IX=2,NINA
      DO 40 JY=1,LINA
      PLOT(IX,JY)=BLANK
      IF(JY.EQ.7.OR.JY.EQ.13.OR.JY.EQ.19.OR.JY.EQ.25.OR.JY.EQ.31.OR.JY.
     1EQ.37.OR.JY.EQ.43.OR.JY.EQ.49.OR.JY.EQ.55)PLOT(IX,JY)=DASH
      IF(JY.EQ.1.OR.JY.EQ.LINA)PLOT(IX,JY)=STAR
   40 CONTINUE
      DO 50 JY=1,LINA
      PLOT(1,JY) =STAR
      IF(JY.EQ.1.OR.JY.EQ.LINA) GO TO 50
      DO 60 IX=11,NEENA,10
   60 PLOT(IX,JY)=AI
   50 PLOT(MINA,JY)=STAR
   70 IF(NXPLOT.EQ.1)GO TO 80
      XA=XD(1)
      XB=XD(1)
      YA=YD(1)
      YB=YD(1)
      DO 90 J=1,N
      IF(XD(J).GT.XA)XA=XD(J)
      IF(XD(J).LT.XB) XB=XD(J)
      IF (YD(J).GT.YA) YA=YD(J)
```

```
 90    IF(YD(J).LT.YB) YB=YD(J)
 80    SCALEX=FLOAT(MINA-3)/(XA-XB)
       SCALEY=FLOAT(LINA-3)/(YA-YB)
       DO 100 J=1,N
       IX=(XD(J)-XB)*SCALEX+2.5
       JY=FLOAT(LINA-1)-(YD(J)-YB)*SCALEY+0.5
       IF(PLOT(IX,JY).EQ.DOT)PLOT(IX,JY)=OVER
       IF(PLOT(IX,JY).EQ.OVER) GO TO 100
       PLOT(IX,JY)=DOT
100    CONTINUE
       SCALEX=9.8/SCALEX
       SCALEY=5.88/SCALEY
       PRINT 120
       KINA=LINA/2
       DO 131 JY=1,LINA
       IF(JY.EQ.KINA)PRINT 150,XB,XA,YB,YA,SCALEX,SCALEY
       IF(JY.EQ.KINA.OR.JY.EQ.KINA+1.OR.JY.EQ.KINA+2.OR.JY.EQ.KINA+3)
      1 GO TO 131
130    PRINT 140,(PLOT(IX,JY),IX=1,MINA)
131    CONTINUE
160    IF(NXPLCT.EQ.1) GO TO 2
10     FORMAT(/10X,*YOUR SLIP IS SHOWING,SIZE EXCEEDS ALLOWABLE LIMITS*)
140    FORMAT(10X121A1)
120    FORMAT(1H1)
150    FORMAT( 41X,*X AXIS MIN=*,1PE12.4,10X,*XMAX=*,1PE12.4/41X,*Y AXIS
      1MIN=*,1PE13.4,10X*YMAX=*,1PE13.4/41X,*HORIZONTAL SCALE - 10  COLUM
      2NS=*,1PE12.2,*  UNITS*,/41X,*VERTICAL SCALE - 6 LINES          =*,
      31PE10.2,*  UNITS*)
       RETURN
       END
C
       SUBROUTINE GRAPHM(  X,  Y,  NS, NP)
C***********************************************************************
C **** EXPLANATION OF FORMAL PARAMETERS ***                  **********
C ****     X      -  HORIZONTAL COORDINATE
C ****     Y      -  VERTICAL COORDINATE
C ****     NS     -  TOTAL NO. OF SETS,NS .LE. 20
C ****     NP(K)  -  NO. OF POINTS IN SET K, K .LE. NS
C **** DIMENSIONS***
C ****     X(FIRST,SECOND), Y(FIRST,SECOND), XD(SECOND), YD(SECOND),
C ****     XMIN(FIRST), XMAX(FIRST), YMIN(FIRST), YMAX(FIRST), NP(FIRST)
C ****     FIRST  -  MAXIMUM NO. OF SETS.
C ****     SECOND -  MAXIMUM NO. OF POINTS IN ANY SET.
C **** EXPLANATION OF AUXILIARY PARAMETERS ***
C ****     XD,YD  -  DUMMY ONE DIMENSIONAL VARIABLES.
C ****     IX,JY  -  HORIZONTAL AND VERTICAL DISTANCES.
C ****     MINA   -  HORIZONTAL SIZE IN NO. OF COLUMNS.
C ****     LINA   -  VERTICAL SIZE IN NO. OF ROWS.
C **** CODES ***
C ****     ISCALE -  0 SAME VERTICAL SCALE FOR ALL SETS.
C ****     ISCALE -  1 DIFFERENT VERTICAL SCALES.
C ****     NPLCT  -  0 GRAPH OF FULL NORMAL SIZE
C ****     NPLCT  -1  GRAPH OF ANY SIZE. (IHSIZE,JVSIZE)
C ****     NPLCT  -2  GRAPHS OF BOTH SIZES
C ****     NOPRNT -     DETAILED LISTING OF MIN * MAX VALUES
```

```
C ****    NOPRNT - 1 SUPPRESS DETAIL LISTING.
C ****    IHSIZE -  HORIZONTAL SIZE IN NO. OF INCHES(INTEGER).LE.13
C ****    JVSIZE -  VERTICAL SIZE IN NO. OF INCHES (INTEGER).LE.10
C ****    (IF NPLOT .EQ. 0  THE LAST TWO WILL NOT BE USED)
C **** THE VALUES OF THESE CODES SHOULD BE TRANSMITTED FROM THE CALLING
C **** PROGRAM TO THE SUBROUTINE THROUGH -
C ****    COMMON /GPHM/ ISCALE,NPLOT,NOPRNT,IHSIZE,JVSIZE
C*****************************************************************************
      DIMENSION XD(120),YD(120),X(5,120),Y(5,120),NP(20),
     1XMAX(20),XMIN(20),YMAX(20),YMIN(20),PT(20),OVER(10),PLOT(131,61)
      DATA BLANK,DOT,STAR,AI,DASH/1H ,1H.,1H*,1HI,1H-/,
     2PT/1H.,1H*,1H+,1HX,1HY,1HF,1HG,1HH,1HJ,1HK,1HL,1HM,1HN,1HP,1HQ,
     31HR,1HS,1HT,1HU,1HV/,
     4OVER/1H1,1H2,1H3,1H4,1H5,1H6,1H7,1H8,1H9,1H0/
       COMMON/GPHM/ISCALE,NPLOT,NOPRNT,IHSIZE,JVSIZE
         IF(IHSIZE.GT.13) IHSIZE=13
      IF(JVSIZE.GT.10)  JVSIZE=10
       NXPLOT=0
       IF (ISCALE .EQ. 1 )  NOPRNT = 1
       IF(NPLOT. EQ. 1)  GO TO 122
       MINA = 131
       LINA = 61
       GO  TO  222
  122  MINA = IHSIZE * 10 + 1
       LINA = JVSIZE * 6 + 1
  222  IF ( NOPRNT .EQ. 1  )  GO  TO  322
       IF ( ISCALE .EQ. 0)   PRINT 981
       IF ( ISCALE .EQ.  1)    GO TO 322
       PRINT 982
  322  MINA = MINA - 1
       NEENA =        MINA - 10
C **** GENERATING THE GRID.
                  DO 11 IX  =  2,MINA
                  DO 11 JY =  1, LINA
       PLOT(IX,JY) = BLANK
       IF ( JY.EQ.7 .OR. JY .EQ.13 .OR. JY .EQ. 19 .OR. JY .EQ. 25 .OR.
      1JY.EQ. 31 .OR. JY .EQ. 37 .OR. JY .EQ. 43 .OR. JY .EQ.49 .OR. JY
      2.EQ. 55 ) PLOT(IX,JY) = DASH
       IF ( JY.EQ.1 .OR. JY .EQ.LINA)PLOT(IX,JY) = STAR
   11             CONTINUE
                  DO 21  JY  = 1, LINA
      PLOT ( 1,JY)  =  STAR
       IF (JY .EQ.1 .OR. JY .EQ. LINA )  GO  TO 21
                  DO 211   IX = 11,NEENA, 10
  211  PLOT(IX,JY) =        AI
   21  PLOT(MINA,JY) = STAR
C **** IF NPLOT IS 1 ONLY REDUCED SIZE IS TO BE PRINTED.
C **** IF NXPLOT IS 1 BOTH SIZES ARE TO BE PRINTED.
  522  IF ( NXPLOT.EQ. 1)  GO TO 622
C **** FINDING MINIMUM AND MAXIMUM FOR EACH SET.
                  DO 61 K =  1,NS
       NPK = NP(K)
                  DO 611 J =  1,NPK
       XD(J) = X(K,J)
  611  YD(J) = Y(K,J)
```

```
      CALL MAXMIN ( XD, NPK, XMAX(K), XMIN(K) )
      CALL MAXMIN ( YD, NPK, YMAX(K), YMIN(K) )
 61   C O N T I N U E
C **** FINDING OVERALL MAX AND MIN FOR ALL VALUES OF X AND Y .
      XA = XMAX(1)
      XB = XMIN(1)
      YA = YMAX(1)
      YB = YMIN(1)
                DO 71 K = 1,NS
      IF (XA.LT.XMAX(K)) XA = XMAX(K)
      IF (YA.LT.YMAX(K)) YA = YMAX(K)
      IF (XB.GT. XMIN(K)) XB = XMIN(K)
 71   IF (YB.GT. YMIN(K)) YB = YMIN(K)
 622  SCALEX = FLOAT(MINA-3) / (XA-XB)
      SCALEY = FLOAT(LINA-3) / (YA-YB)
                DO 81 K = 1,NS
      NPK = NP(K)
                DO 811 J =  1,NPK
      XD(J)=X(K,J)
 811  YD(J)=Y(K,J)
      IF (ISCALE.EQ.0) GO TO 32
      YA=YMAX(K)
      YB=YMIN(K)
      SCALEY=FLOAT(LINA-3)/(YA-YB)
C **** FINDING THE POINT ON THE GRAPH.
 32   DO 821 J=1,NPK
      IX=(XD(J)-XB)*SCALEX+2.5
      JY=FLOAT(LINA-1)-(YD(J)-YB)*SCALEY+2.5-2.
      IF(K.EQ.1) GO TO 42
      DO 8211 L=2,9
      IF(PLOT(IX,JY).EQ.OVER(L)) GO TO 52
 8211 CONTINUE
      GO TO 62
 52   PLOT(IX,JY)=OVER(L+1)
      GO TO 821
 62   KM=K-1
      DO 8221 L=1,KM
 8221 IF(PLOT(IX,JY).EQ.PT(L)) GO TO 22
      GO TO 42
 22   PLOT(IX,JY)=OVER(2)
      GO TO 821
 42   PLOT(IX,JY)=PT(K)
 821  CONTINUE
      IF(NOPRNT.EQ.1) GO TO 81
      IF(ISCALE.EQ.0) GO TO 72
      SCALEY=5.88/SCALEY
      PRINT 984,K,XMIN(K),XMAX(K),YMIN(K),YMAX(K),SCALEY,PT(K)
      GO TO 81
      PRINT 983,K,XMIN(K),XMAX(K),YMIN(K),YMAX(K),PT(K)
      CONTINUE
      SCALEY=5.88/SCALEY
      SCALEX=9.8/SCALEX
      IF(NOPRNT.EQ.1) GO TO 92
      IF(ISCALE.EQ.0) PRINT 985,SCALEX,SCALEY
      IF(ISCALE.EQ.1) PRINT 986,SCALEX
```

```
82     IF(NOPRNT.EQ.1)  PRINT 986
C **** PRINT THE GRAPH
       KINA=LINA/2
       DO 102    JY=1,LINA
       IF(JY.EQ.KINA)  GO TO 103
       GO TO 104
103    IF(NOPRNT.EQ.1)PRINT989,XB,XA,YB,YA,SCALEX,SCALEY
104    IF(JY.EQ.KINA.OR.JY.EQ.KINA+1.OR.JY.EQ.KINA+2.OR.JY.EQ.KINA+3)
     1   GO TO 102
101    PRINT 987,(PLOT(IX,JY),IX=1,MINA)
102    CONTINUE
C **** TEST IF GRAPH OF REDUCED SIZE IS TO BE PRINTED.
       IF(NPLOT.EQ.2)  NXPLOT=NXPLOT+1
       IF(NOPRNT.EQ.1)  GO TO 92
       IF(NPLOT.EQ.1.OR.NXPLOT.EQ.2) PRINT 388
92     IF(NXPLCT.EQ.1)  GO TO 122
981    FORMAT(2H1,120(1H*)/2H *118X1H*/2H *9X*SET*11X*X (MIN)*13X*X (MA
     2*13X*Y (MIN)*13X*Y (MAX)*12X*LETTER*10X1H*  )
982    FORMAT(1H1,120(1H*)/1X1H*118X1H*/2H *6X*SET*10X*X (MIN)*12X*X (M
     2)*12X*Y (MIN)*12X*Y (MAX)*12X*Y SCALE*6X*LETTER*4X1H*  )
983    FORMAT(2H *118X1H*/2H *9XI2,8X4(1PE12.4,8X),6XA1,1X1H*)
984    FORMAT(2H *118X1H*/2H *6XI2,7X4(1PE12.4,7X)1PE12.2,8XA1,6X1H*)
985    FORMAT(2H *118X1H*/2H *27X*HORIZONTAL SCALE  -  10 COLUMNS = *,
     2 1PE12.2,*  UNITS*38X1H*/2H *118X1H*/2H *27X*VERTICAL SCALE  -
     3 6 LINES  = *1PE12.2,*  UNITS*,38X1H*/2H *118X1H*/1X120(1H*)/1H1
986    FORMAT(2H *118X1H*/2H *27X*HORIZONTAL SCALE  -  10 COLUMNS = *,
     2 1PE12.2,*  UNITS*33X1H*/2H *118X1H*/2H *10X*VERTICAL SCALES ARE
     3IFFERENT CORRESPONDING TO MIN AND MAX VALUES OF Y CO-ORDINATE IN
     4ACH SET*14X1H*/2H *47X*(6 LINES = YY UNITS)*51X1H*/2H *118X1H*/
     5 1X120(1H*)/1H1  )
987    FORMAT(1 X121A1)
388    FORMAT(////////////    )
988    FORMAT(1H )
989    FORMAT(40X*X AXIS MIN = *1PE12.4,10X*MAX = *1PE12.4/40X*Y AXIS M
     2 =*1PE13.4,10X*MAX =*1PE13.4/4 X*HORIZONTAL SCALE - 10 COLUMNS =
     31PE12.2,*  UNITS*/40X*VERTICAL SCALE  -  6 LINES  =*1PE12.2,
     4*  UNITS*)
       RETURN
       END
C
       SUBROUTINE MAXMIN (XD,N,XMAX,XMIN )
       DIMENSICN XD(120)
       XMAX=XD(1)
       XMIN=XMAX
       DO 51 J=1,N
       IF (XD(J).GT. XMAX ) XMAX= XD(J)
       IF (XD(J) .LT. XMIN ) XMIN= XD(J)
       CONTINUE
       RETURN
       END
```

APPENDIX 'F'

TYPICAL INPUT/OUTPUT

CESSNA 182

MASS AND GEOMETRICAL CHARACTERISTICS

WEIGHT (LBS)	MOMENT OF INERTIA (SLUGS SQ.FT) IXX	IZZ	SPAN (FT)	WING AREA (SQ.FT)
[illegible]	[illegible].0	1967.0	36.08	174.35

FLIGHT [illegible]

[illegible]	DENSITY (SLUGS/CU.FT)	LIFT COEFFICIENT	ANGLE OF ATTACK(DEG)	FLIGHT PATH ANGLE(DEG)
[illegible]	0.[illegible]72	1.[illegible]5	[illegible].2[illegible]	-6.8[illegible]

STABILITY [illegible]

[illegible]	ROLL RATE	YAW RATE	CONTROL
[illegible]	CYP=-0.2124	CYR= 0.2010	CYC= 0.0000
[illegible]	CLP=-0.4940	CLR= 0.2039	CLC= [illegible].0000
[illegible]	CNP=-[illegible].[illegible]96	CNR=-[illegible].1151	CNC= [illegible].0000

INITIAL [illegible]

ANGLES(DEG)		RATES(DEG/SEC)	
ROLL	[illegible]	ROLL	0.0
YAW	[illegible]	YAW	0.0
SIDESLIP	10.0		

VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL

AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
0.0000	0.0000	0.0000	10.0000	0.0000	0.0000
0.0500	-0.0997	0.0484	9.9117	-3.7004	1.9172
0.1000	-0.3465	0.1896	9.7306	-5.9699	3.7079
0.1500	[illegible]	0.4169	9.4613	-7.2278	5.3589
0.2000	[illegible]	0.7230	9.1096	-7.7694	6.8595
0.2500	-1.4486	1.1002	8.6828	-7.8037	8.2017
0.3000	[illegible]	1.5404	8.1888	-7.4799	9.3797
0.3500	[illegible]	2.0354	7.6360	-6.9053	10.3898
0.4000	-2.5196	2.5766	7.0333	-6.1583	11.2305
0.4500	-2.8064	3.1556	6.3897	-5.2970	11.9024
0.5000	-3.0482	3.7640	5.7142	-4.3648	12.4376
0.5500	[illegible]	4.3936	5.0156	-3.3951	12.7502
0.6000	[illegible]	5.0354	4.3027	-2.4138	12.9356
0.6500	[illegible]	5.6847	3.5838	-1.4414	12.9707
0.7000	[illegible]	6.3311	2.8670	-0.4941	12.8637
0.7500	-3.5549	6.9689	2.1597	0.4149	12.6235
0.8000	[illegible]	7.5915	1.4591	1.2743	12.2607
0.8500	-3.4014	8.1931	0.8014	2.0774	11.7857
0.9000	[illegible]	8.7683	0.1527	2.8149	11.21[illegible]
0.9500	[illegible]	9.3126	-0.4420	3.4831	10.5456
1.0000	-2.9[illegible]77	9.8216	-1.0080	4.0776	9.8046
1.0500	[illegible]	10.2920	-1.5316	4.5958	8.9992
1.1000	[illegible]	10.7207	-2.0095	5.0361	8.1418
1.1500	[illegible]	11.1055	-2.4391	5.3980	7.2444
1.2000	[illegible]	11.4447	-2.8186	5.6818	6.3189
1.2500	[illegible]	11.7371	-3.1468	5.8888	5.3769
1.3000	[illegible]	11.9823	-3.4230	6.0210	4.4294
1.3500	[illegible]	12.1801	-3.6473	6.0813	3.4869
1.4000	-0.7465	12.3312	-3.8202	6.0730	2.5590
1.4500	-0.4442	12.4364	-3.9426	6.0001	1.6551

VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
1.5000	-[illegible]473	[illegible].4972	-4.[illegible]163	5.8669	[illegible].7834
1.5500	[illegible]	[illegible].5154	-4.[illegible]431	5.6784	-0.0486
1.5000	0.[illegible]97	[illegible].4932	-4.0254	5.4396	-0.8342
1.5500	[illegible]	[illegible].4329	-3.966[illegible]	5.[illegible]559	-1.5676
1.7[illegible]	[illegible]	[illegible].5373	-3.8677	4.833[illegible]	-2.2439
1.75[illegible]	[illegible]	[illegible].2[illegible]95	-3.7339	4.4765	-2.859[illegible]
1.8000	[illegible]	[illegible].0525	-3.5681	4.092[illegible]	-3.41[illegible]
1.85[illegible]	[illegible]	[illegible].6596	-3.3737	3.6855	-3.8945
1.9[illegible]	1.75[illegible]	[illegible].6642	-3.1546	3.2623	-4.311[illegible]
1.95[illegible]	[illegible]	[illegible].4296	-2.9144	2.828[illegible]	-4.6590
2.0000	[illegible]	[illegible].1994	-2.6569	2.388[illegible]	-4.9386
2.05[illegible]	[illegible]	[illegible].9469	-2.3859	1.9472	-5.15[illegible]6
2.1[illegible]	[illegible]	[illegible].5855	-2.[illegible]51	1.51[illegible]4	-5.2966
2.15[illegible]	[illegible]	[illegible].4185	-1.8179	1.0820	-5.3789
2.2000	[illegible]	[illegible].1486	-1.5278	0.6662	-5.40[illegible]
2.2500	[illegible]	9.6793	-1.2382	[illegible].2667	-5.3533
2.3000	2.[illegible]6	[illegible].5132	-[illegible].9523	-[illegible].113[illegible]	-5.2725
2.3500	2.[illegible]	9.[illegible]529	-0.6722	-0.470[illegible]	-5.1314
2.4000	2.[illegible]	9.[illegible]008	-0.4013	-0.802[illegible]	-4.9446
2.4500	2.[illegible]	[illegible].8591	-[illegible].1419	-1.1[illegible]71	-4.7166
2.5000	2.[illegible]362	[illegible].5297	[illegible].104[illegible]	-1.3828	-4.4522
2.5500	2.[illegible]308	[illegible].4144	0.3346	-1.6282	-4.1563
2.5000	2.[illegible]419	[illegible].2146	[illegible].5481	-1.8423	-3.8337
2.55[illegible]	1.947[illegible]	8.[illegible]314	[illegible].7434	-2.[illegible]247	-3.4896
2.7[illegible]	1.842[illegible]	7.8659	[illegible].9192	-2.1751	-3.1287
2.7500	1.720[illegible]	7.7137	1.0750	-2.2938	-2.7560
2.8000	1.6[illegible]	7.5904	1.2100	-2.3813	-2.3760
2.35[illegible]	1.4975	7.4812	1.3240	-2.4384	-1.9933
2.9000	1.3698	7.3910	1.4171	-2.4662	-1.6121
2.9500	1.[illegible]463	7.3198	1.4894	-2.4661	-1.2364

VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL

AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
3.0000	1.1236	7.2672	1.5413	-2.4397	-0.8703
3.0500	[illegible]	7.2326	1.5733	-2.3888	-0.5151
3.1000	0.8851	7.2154	1.5864	-2.3152	-0.1780
3.1500	0.7716	7.2145	1.5813	[illegible]	0.1417
3.2000	0.6543	7.2292	1.5593	[illegible]	0.4416
3.2500	[illegible]	7.2582	1.5214	-1.9799	0.7166
3.3000	0.4555	7.3004	1.4690	-1.8374	0.9581
3.3500	0.3775	7.3546	1.4035	-1.6833	1.1939
3.4000	[illegible]	7.4194	1.3264	[illegible]	1.3978
3.4500	0.2256	7.4934	1.2390	-1.3499	1.5644
3.5000	0.1624	7.5753	1.1430	-1.1749	1.7083
3.5500	[illegible]	7.6638	1.0398	-0.9974	1.8245
3.6000	[illegible]	7.7573	0.9309	-0.8193	1.9134
3.6500	[illegible]	7.8547	0.8179	-0.6426	1.9756
3.7000	[illegible]	7.9545	0.7022	-0.4691	2.0119
3.7500	-0.0203	8.0555	0.5852	[illegible]	2.0234
3.8000	[illegible]	8.1564	0.4681	-0.1383	2.0114
3.8500	[illegible]	8.2562	0.3524	[illegible]	1.9771
3.9000	[illegible]	8.3535	0.2390	0.1515	1.9227
3.9500	[illegible]	8.4482	0.1292	0.2968	1.8495
4.0000	[illegible]	8.5385	0.0239	0.4213	1.7594
4.0500	[illegible]	8.6239	-0.0761	0.5340	1.6545
4.1000	[illegible]	8.7037	-0.1699	0.6345	1.5363
4.1500	[illegible]	8.7773	-0.2570	0.7226	1.4073
4.2000	[illegible]	8.8443	-0.3367	0.7977	1.2694
4.2500	0.1659	8.9041	-0.4086	0.8600	1.1245
4.3000	0.2102	8.9566	-0.4725	0.9095	0.9745
4.3500	0.2567	9.0015	-0.5280	0.9463	0.8214
4.4000	0.3046	9.0388	-0.5750	0.9708	0.6670
4.4500	0.3536	9.0683	-0.6136	0.9834	0.5130

VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL

AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
4.5000	[illegible]	9.0901	-0.5437	0.9847	0.3611
4.5500	[illegible]	[illegible]	-0.6556	0.9752	0.2125
4.6000	[illegible]	[illegible]	-0.5794	0.9556	0.0691
4.6500	[illegible]	[illegible]	-0.5555	0.9271	-0.0581
4.7000	[illegible]	9.1045	-0.5541	[illegible]	[illegible]
4.7500	[illegible]	[illegible]	-0.5762	[illegible]	[illegible]
4.8000	[illegible]	[illegible]	-0.5616	[illegible]	[illegible]
4.8500	[illegible]	9.0467	-0.5411	0.7942	[illegible]
4.9000	[illegible]	[illegible]	-0.6152	0.7373	[illegible]
4.9500	[illegible]	8.9852	-0.5845	0.6755	[illegible]
5.0000	[illegible]	[illegible]	-0.5497	[illegible]	[illegible]
5.0500	[illegible]	8.9085	-0.5112	0.5416	-0.7791
5.1000	[illegible]	8.8651	-0.4698	0.4712	[illegible]
5.1500	[illegible]	[illegible]	-0.4251	0.3996	-0.8857
5.2000	[illegible]	8.7751	-0.3806	0.3275	-0.9238
5.2500	0.9017	8.7252	-0.3359	0.2562	-0.9500
5.3000	[illegible]	8.6767	-0.2867	[illegible]	-0.9653
5.3500	[illegible]	8.6282	-0.2394	[illegible]	[illegible]
5.4000	0.9155	8.5801	-0.1925	-0.0110	-0.9575
5.4500	0.9135	8.5328	-0.1466	[illegible]	-0.9548
5.5000	0.9085	8.4868	-0.1120	-0.1253	-0.9336
5.5500	0.9019	8.4425	-0.0592	-0.1761	-0.9048
5.6000	0.8910	8.4000	-0.0185	-0.2222	-0.8691
5.6500	0.8789	8.3598	0.0197	-0.2634	-0.8272
5.7000	0.8648	8.3221	0.0552	-0.2995	-0.7800
5.7500	0.8493	8.2871	0.0878	-0.3305	-0.7283
5.8000	0.8318	8.2549	0.1173	-0.3562	-0.6729
5.8500	0.8135	8.2256	0.1435	-0.3768	-0.6146
5.9000	0.7942	8.1995	0.1663	-0.3922	-0.5541
5.9500	0.7743	8.1764	0.1857	-0.4027	-0.4923
-0.4299					

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MASS AND GEOMETRICAL CHARACTERISTICS

WEIGHT (LBS)	MOMENT OF INERTIA (SLUGS SQ.FT) IXX	IZZ	SPAN (FT)	WING AREA (SQ.FT)
2650.0	948.0	1967.0	36.08	174.38

FLIGHT CONDITIONS

VELOCITY (MPH)	DENSITY (SLUGS/CU.FT)	LIFT COEFFICIENT	ANGLE OF ATTACK(DEG)	FLIGHT PATH ANGLE(DEG)
73.0	0.002378	1.115	10.20	-6.80

STABILITY DERIVATIVES

SIDESLIP	ROLL RATE	YAW RATE	CONTROL
CYB=-0.3191	CYP= 0.0304	CYR= 0.1714	CYC= 0.0000
CLB=-0.0919	CLP=-0.4633	CLR= 0.2654	CLC= 0.0000
CNB= 0.0580	CNP=-0.1353	CNR=-0.1254	CNC= 0.0000

INITIAL CONDITIONS

ANGLES(DEG)		RATES(DEG/SEC)	
ROLL	0.0	ROLL	0.0
YAW	0.0	YAW	0.0
SIDESLIP	10.0		

VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
0.0000	0.0000	0.0000	10.0000	0.0000	0.0000
0.0500	-0.0941	0.0417	9.9151	-3.5007	1.6589
0.1000	-0.3280	0.1646	9.7473	-5.6642	3.2440
0.1500	-0.6446	0.3644	9.4995	-6.8600	4.7251
0.2000	-1.0025	0.6351	9.1770	-7.3578	6.0808
0.2500	-1.3721	0.9701	8.7861	-7.3551	7.2960
0.3000	-1.7322	1.3622	8.3343	-6.9974	8.3614
0.3500	-2.0678	1.8036	7.8295	-6.3927	9.2715
0.4000	-2.3687	2.2867	7.2802	-5.6216	10.0245
0.4500	-2.6282	2.8035	6.6949	-4.7447	10.6213
0.5000	-2.8422	3.3463	6.0818	-3.8080	11.0651
0.5500	-3.0086	3.9075	5.4493	-2.8467	11.3611
0.6000	-3.1269	4.4800	4.8053	-1.8876	11.5160
0.6500	-3.1977	5.0569	4.1574	-0.9514	11.5378
0.7000	-3.2227	5.6317	3.5126	-0.0543	11.4355
0.7500	-3.2040	6.1986	2.8775	0.7917	11.2190
0.8000	-3.1445	6.7519	2.2581	1.5773	10.8986
0.8500	-3.0474	7.2869	1.6598	2.2957	10.4853
0.9000	-2.9162	7.7991	1.0875	2.9421	9.99[illegible]0
0.9500	-2.7544	8.2847	0.5453	3.5135	9.4240
1.0000	-2.5661	8.7405	0.0368	4.0081	8.7985
1.0500	-2.3549	9.1637	-0.4351	4.4256	8.1245
1.1000	-2.1248	9.5523	-0.8680	4.7666	7.4125
1.1500	-1.8795	9.9045	-1.2601	5.0327	6.6731
1.2000	-1.6227	10.2193	-1.6103	5.2262	5.9161
1.2500	-1.3580	10.4960	-1.9179	5.3502	5.1508
[illegible]	-1.0888	10.7344	-2.1826	5.4083	4.3860
1.3500	-0.8182	10.9347	-2.4049	5.4047	3.6299
1.4000	-0.5493	11.0977	-2.5855	5.3437	2.8898
1.4500	-0.2847	11.2241	-2.7254	5.2302	2.1725

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VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
1.5000	-0.0271	11.3154	-2.8262	5.0692	1.4840
1.5500	0.2215	11.3731	-2.8897	4.8659	0.8296
1.6000	0.4589	11.3990	-2.9178	4.6256	0.2136
1.6500	0.6835	11.3951	-2.9130	4.3534	-0.3600
1.7000	0.8938	11.3637	-2.8776	4.0546	-0.8883
1.7500	1.0866	11.3071	-2.8143	3.7343	-1.3691
1.8000	1.2670	11.2277	-2.7258	3.3976	-1.8005
1.8500	1.4282	11.1279	-2.6148	3.0492	-2.1816
1.9000	1.5718	11.0103	-2.4843	2.6937	-2.5121
1.9500	1.6975	10.8775	-2.3370	2.3354	-2.7921
2.0000	[illegible]	10.7320	-2.1757	1.9783	-3.0223
2.0500	1.8954	10.5761	-2.0032	1.6262	-3.2038
2.1000	1.9681	10.4124	-1.8221	1.2824	-3.3385
2.1500	2.0238	10.2430	-1.6351	0.9501	-3.4281
2.2000	2.0633	10.0703	-1.4444	0.6319	-3.4750
2.2500	[illegible]	9.8962	-1.2526	0.3302	-3.4820
2.3000	2.0966	9.7227	-1.0615	0.0469	-3.4517
2.3500	2.0923	9.5516	-0.8734	-0.2161	-3.3874
2.4000	[illegible]	9.3845	-0.6898	-0.4577	-3.2921
2.4500	2.0469	9.2228	-0.5126	-0.6767	-3.1691
2.5000	2.0081	9.0680	-0.3431	-0.8725	-3.0218
2.5500	1.9601	8.9210	-0.1825	-1.0448	-2.8535
2.6000	1.9040	8.7829	-0.0319	-1.1933	-2.6676
2.6500	1.8411	8.6545	0.1078	-1.3183	-2.4673
2.7000	1.7726	8.5364	0.2360	-1.4201	-2.2558
2.7500	1.6995	8.4290	0.3520	-1.4993	-2.0363
2.8000	1.6230	8.3328	0.4556	-1.5568	-1.8115
2.8500	1.5442	8.2479	0.5465	-1.5935	-1.5844
2.9000	1.4640	8.1744	0.6247	-1.6106	-1.3575
2.9500	1.3834	8.1121	0.6903	-1.6092	-1.1332

VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
3.0000	1.3033	8.0610	0.7436	-1.5909	-0.9138
3.0500	1.2246	8.0206	0.7848	-1.5569	-0.7011
3.1000	1.1479	7.9907	0.8144	-1.5089	-0.4970
3.1500	1.0739	7.9708	0.8330	-1.4483	-0.3031
3.2000	1.0032	7.9602	0.8411	-1.3767	-0.1206
3.2500	0.9364	7.9585	0.8394	-1.2957	0.0493
3.3000	0.8738	7.9649	0.8286	-1.2069	0.2057
3.3500	0.8158	7.9788	0.8096	-1.1117	0.3481
3.4000	0.7627	7.9995	0.7832	-1.0117	0.4756
3.4500	0.7147	8.0261	0.7501	-0.9082	0.5883
3.5000	0.6719	8.0580	0.7112	-0.8027	0.6860
3.5500	0.6345	8.0945	0.6674	-0.6964	0.7587
3.6000	0.6023	8.1347	0.6194	-0.5904	0.8366
3.6500	0.5754	8.1779	0.5682	-0.4860	0.8901
3.7000	0.5536	8.2234	0.5144	-0.3841	0.9297
3.7500	0.5369	8.2706	0.4589	-0.2856	0.9563
3.8000	0.5253	8.3188	0.4025	-0.1912	0.9596
3.8500	0.5177	8.3674	0.3453	-0.1018	0.9714
3.9000	0.5147	8.4158	0.2887	-0.0179	0.9621
3.9500	0.5156	8.4635	0.2329	0.0599	0.9428
4.0000	0.5206	8.5099	0.1785	0.1314	0.9143
4.0500	0.5288	8.5547	0.1259	0.1963	0.8776
4.1000	0.5401	8.5976	0.0757	0.2542	0.8337
4.1500	0.5542	8.6380	0.0281	0.3051	0.7837
4.2000	0.5705	8.6758	-0.0165	0.3490	0.7284
4.2500	0.5889	8.7108	-0.0578	0.3859	0.6688
4.3000	0.6090	8.7427	-0.0958	0.4159	0.6060
4.3500	0.6304	8.7713	-0.1301	0.4393	0.5408
4.4000	0.6528	8.7967	-0.1607	0.4562	0.4741
4.4500	0.6759	8.8187	-0.1876	0.4669	0.4068

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VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
4.5000	0.6994	8.8374	-0.2107	0.4718	0.3394
4.5500	0.7230	8.8527	-0.2301	0.4713	0.2729
4.6000	0.7465	8.8647	-0.2458	0.4657	0.2079
4.6500	0.7695	8.8735	-0.2579	0.4555	0.1448
4.7000	0.7919	8.8792	-0.2666	0.4411	0.0844
4.7500	0.8136	8.8820	-0.2720	0.4231	0.0269
4.8000	0.8342	8.8820	-0.2743	0.4017	-0.0271
4.8500	0.8537	8.8794	-0.2737	0.3776	-0.0774
4.9000	0.8719	8.8743	-0.2704	0.3512	-0.1237
4.9500	0.8888	8.8671	-0.2647	0.3229	-0.1658
5.0000	0.9042	8.8578	-0.2568	0.2932	-0.2035
5.0500	0.9181	8.8468	-0.2470	0.2625	-0.2368
5.1000	0.9304	8.8342	-0.2354	0.2312	-0.2657
5.1500	0.9412	8.8203	-0.2223	0.1996	-0.2901
5.2000	0.9504	8.8053	-0.2081	0.1682	-0.31[illegible]
5.2500	0.9580	8.7894	-0.1928	0.1372	-0.3259
5.3000	0.9641	8.7728	-0.1768	0.1070	-0.3375
5.3500	0.9687	8.7557	-0.1603	0.0778	-0.3451
5.4000	0.9719	8.7383	-0.1435	0.0498	-0.3491
5.4500	0.9738	8.7208	-0.1266	0.0234	-0.3495
5.5000	0.9743	8.7034	-0.1098	-0.0015	-0.3466
5.5500	0.9736	8.6862	-0.0932	-0.0245	-0.3408
5.6000	0.9719	8.6694	-0.0771	-0.0457	-0.3323
5.6500	0.9691	8.6530	-0.0615	-0.0649	-0.3213
5.7000	0.9654	8.6373	-0.0466	-0.0820	-0.3082
5.7500	0.9609	8.6222	-0.0325	-0.0971	-0.2933
5.8000	0.9557	8.6080	-0.0193	-0.1100	-0.2768
5.8500	0.9530	8.5946	-0.0071	-0.1209	-0.2591
5.9000	0.9437	8.5821	0.0042	-0.1298	-0.2404
5.9500	0.9370	8.5705	0.0143	-0.1367	-0.2211

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CESSNA 182

MASS AND GEOMETRICAL CHARACTERISTICS

WEIGHT (LBS)	MOMENT OF INERTIA (SLUGS SQ.FT) IXX	IZZ	SPAN (FT)	WING AREA (SQ.FT)
2650.0	948.0	1967.0	36.08	174.38

FLIGHT CONDITIONS

VELOCITY (MPH)	DENSITY (SLUGS/CU.FT)	LIFT COEFFICIENT	ANGLE OF ATTACK(DEG)	FLIGHT PATH ANGLE(DEG)
150.0	0.002049	0.306	0.36	0.00

STABILITY DERIVATIVES

SIDESLIP	ROLL RATE	YAW RATE	CONTROL
CYB=-0.3030	CYP=-0.0752	CYR= 0.2140	CYC= 0.0000
CLB=-0.0923	CLP=-0.4840	CLR= 0.0798	CLC= 0.0000
CNB= 0.0587	CNP=-0.0278	CNR=-0.0937	CNC= 0.0000

INITIAL CONDITIONS

ANGLES(DEG)		RATES(DEG/SEC)	
ROLL	0.0	ROLL	0.0
YAW	0.0	YAW	0.0
SIDESLIP	10.0		

VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL
AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
0.0000	0.0000	0.0000	10.0000	0.0000	0.0000
0.0500	-0.3062	0.1164	9.8129	-10.8556	4.6245
0.1000	-0.9919	.4574	9.4026	-15.8119	8.9541
0.1500	-1.8350	1.0043	8.7865	-17.4839	12.8391
0.2000	-2.7101	1.7322	7.9899	-17.2690	16.1793
0.2500	-3.5439	2.6120	7.0425	-15.9334	18.9386
0.3000	-4.2924	3.6122	5.9770	-13.9157	20.9883
0.3500	-4.9288	4.5997	4.8275	-11.4841	22.4027
0.4000	-5.4370	5.8414	3.6283	-8.8170	23.1560
0.4500	-5.8087	7.0046	2.4128	-6.0441	23.2699
0.5000	-6.0414	8.1583	1.2129	-3.2671	22.7817
0.5500	-6.1368	9.2736	0.0581	-0.5695	21.7418
0.6000	-6.1008	10.3243	-1.0252	1.9787	20.2116
0.6500	-5.9424	11.2877	-2.0141	4.3187	18.2610
0.7000	-5.6732	12.1447	-2.8894	6.4026	15.9563
0.7500	-5.3071	12.8800	-3.6362	8.1927	13.4078
0.8000	-4.8593	13.4824	-4.2436	9.6519	10.6671
0.8500	-4.3465	13.9450	-4.7050	10.7929	7.8254
0.9000	-3.7858	14.2646	-5.0181	11.5784	4.9615
0.9500	-3.1944	14.4420	-5.1844	12.0204	2.1495
1.0000	-2.5893	14.4816	-5.2088	12.1295	-0.5425
1.0500	-1.9867	14.3908	-5.1000	11.9240	-3.0537
1.1000	-1.4018	14.1801	-4.8590	11.4292	-5.3319
1.1500	-0.8481	13.8622	-4.5296	10.6762	-7.3339
1.2000	-0.3379	13.4519	-4.0972	9.7003	-9.0265
1.2500	0.1188	12.9651	-3.5888	8.5407	-10.3864
1.3000	0.5138	12.4190	-3.0221	7.2384	-11.4002
1.3500	0.8410	11.8309	-2.4153	5.8357	-12.0642
1.4000	1.0964	11.2183	-1.7862	4.3745	-12.3837
1.4500	1.2782	10.5981	-1.1524	2.8957	-12.3726

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VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL

AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
1.5000	1.3863	9.9862	-0.5301	1.4379	-12.0522
1.5500	1.4229	9.3976	0.0655	0.0370	-11.4502
1.6000	1.3915	8.8454	0.6210	-1.2752	-10.6000
1.6500	1.2973	8.3411	1.1249	-2.4710	-9.5389
1.7000	1.1467	7.8943	1.5675	-3.5273	-8.3072
1.7500	0.9472	7.5125	1.9417	-4.4263	-6.9472
1.8000	0.7070	7.2010	2.2422	-5.1551	-5.5015
1.8500	0.4347	6.9631	2.4660	-5.7061	-4.0123
1.9000	0.1394	6.7998	2.6125	-6.0764	-2.5202
1.9500	-0.1700	6.7105	2.6827	-6.2683	-1.0633
2.0000	-0.4846	6.6923	2.6798	-6.2883	0.3236
2.0500	-0.7961	6.7412	2.6084	-6.1466	1.6098
2.1000	-1.0968	6.8512	2.4748	-5.8574	2.7691
2.1500	-1.3797	7.0156	2.2863	-5.4371	3.7799
2.2000	-1.6386	7.2264	2.0512	-4.9050	4.6261
2.2500	-1.8686	7.4753	1.7784	-4.2816	5.2958
2.3000	-2.0656	7.7531	1.4773	-3.5886	5.7859
2.3500	-2.2267	8.0508	1.1573	-2.8480	6.0927
2.4000	-2.3510	8.3594	0.8277	-2.0817	6.2210
2.4500	-2.4348	8.6701	0.4976	-1.3107	6.1789
2.5000	-2.4813	8.9746	0.1752	-0.5550	5.9782
2.5500	-2.4909	9.2655	-0.1315	0.1673	5.6343
2.6000	-2.4654	9.5359	-0.4159	0.8399	5.1548
2.6500	-2.4079	9.7832	-0.6722	1.4488	4.5899
2.7000	-2.3218	9.9935	-0.8956	1.9827	3.9307
2.7500	-2.2111	10.1722	-1.0826	2.4328	3.2094
2.8000	-2.0800	10.3138	-1.2307	2.7930	2.4483
2.8500	-1.9353	10.4158	-1.3386	3.0598	1.6693
2.9000	-1.7756	10.4808	-1.4052	3.2324	0.8932
2.9500	-1.6116	10.5065	-1.4342	3.3124	0.1397

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VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL
AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
3.0000	-1.4459	10.4954	-1.4246	3.3037	-0.5737
3.0500	-1.2826	10.4500	-1.3799	3.2121	-1.2312
3.1000	-1.1259	10.3735	-1.3035	3.0453	-1.8198
3.1500	-0.9792	10.2694	-1.1993	2.8123	-2.3288
3.2000	-0.8456	10.1420	-1.0719	2.5231	-2.7505
3.2500	-0.7277	9.9959	-0.9258	2.1888	-3.0796
3.3000	-0.6273	9.8357	-0.7661	1.8206	-3.3136
3.3500	-0.5460	9.6661	-0.5975	1.4302	-3.4528
3.4000	-0.4845	9.4919	-0.4253	1.0288	-3.4995
3.4500	-0.4431	9.3176	-0.2532	0.6273	-3.4585
3.5000	-0.42[illegible]6	9.1474	-0.0865	0.2359	-3.3364
3.5500	-0.4192	8.9852	0.0714	-0.1360	-3.1414
3.6000	-0.4347	8.8344	0.2169	-0.4803	-2.8832
3.6500	-0.4665	8.6978	0.3471	-0.7900	-2.5723
3.7000	-0.5[illegible]	8.5778	0.4596	-1.0594	-2.2199
3.7500	-0.5718	8.4763	0.5529	-1.2842	-1.8377
3.8000	-0.64[illegible]7	8.3943	0.6256	-1.4616	-1.4373
3.8500	-0.7172	8.3327	0.6773	-1.5900	-1.0299
3.9000	-0.7989	8.2913	0.7080	-1.6694	-0.6264
3.9500	-0.8833	8.2698	0.7182	-1.7008	-0.2368
4.0000	-0.9682	8.2672	0.7091	-1.6865	0.1299
4.0500	-1.0512	8.2823	0.6820	-1.6297	0.4659
4.1000	-1.1305	8.3132	0.6388	-1.5347	0.7646
4.1500	-1.2042	8.3580	0.5816	-1.4062	1.0208
4.2000	-1.2706	8.4145	0.5127	-1.2496	1.2306
4.2500	-1.3287	8.4803	0.4347	-1.0708	1.3917
4.3000	-1.3775	8.5528	0.3502	-0.8756	1.5032
4.3500	-1.4161	8.6298	0.2616	-0.67[illegible]	1.5653
4.4000	-1.4444	8.7086	0.1715	-0.4601	1.5796
4.4500	-1.4622	8.7870	0.0822	-0.2514	1.5487

VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL
AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
4.5000	-1.4696	8.8628	-0.0039	-0.0490	1.4765
4.5500	-1.4672	8.9340	-0.0850	0.1423	1.3672
4.6000	-1.4557	8.9989	-0.1593	0.3182	1.2262
4.6500	-1.4357	9.0562	-0.2253	0.4754	1.0589
4.7000	-1.4085	9.1045	-0.2818	0.6110	0.8712
4.7500	-1.3750	9.1431	-0.3281	0.7230	0.6694
4.8000	-1.3366	9.1713	-0.3637	0.8100	0.4593
4.8500	-1.2945	9.1889	-0.3882	0.8713	0.2469
4.9000	-1.2499	9.1960	-0.4018	0.9071	0.0377
4.9500	-1.2042	9.1929	-0.4049	0.9181	-0.1631
5.0000	-1.1585	9.1799	-0.3979	0.9056	-0.3511
5.0500	-1.1140	9.1580	-0.3818	0.8714	-0.5222
5.1000	-1.0717	9.1281	-0.3575	0.8177	-0.6732
5.1500	-1.0325	9.0911	-0.3262	0.7471	-0.8015
5.2000	-0.9972	9.0483	-0.2891	0.6626	-0.9053
5.2500	-0.9664	9.0010	-0.2475	0.5670	-0.9835
5.3000	-0.9406	8.9514	-0.2027	0.4637	-1.0357
5.3500	-0.9201	8.8979	-0.1562	0.3557	-1.0623
5.4000	-0.9051	8.8446	-0.1091	0.2460	-1.0642
5.4500	-0.8955	8.7918	-0.0628	0.1375	-1.0430
5.5000	-0.8913	8.7406	-0.0183	0.0329	-1.0006
5.5500	-0.8921	8.6921	0.0233	-0.0653	-0.9395
5.6000	-0.8976	8.6470	0.0612	-0.1552	-0.8624
5.6500	-0.9074	8.6060	0.0946	-0.2349	-0.7722
5.7000	-0.9209	8.5699	0.1230	-0.3030	-0.6722
5.7500	-0.9375	8.5389	0.1460	-0.3587	-0.5654
5.8000	-0.9566	8.5134	0.1635	-0.4012	-0.4551
5.8500	-0.9774	8.4934	0.1749	-0.4303	-0.3442
5.9000	-0.9994	8.4790	0.1808	-0.4461	-0.2355
5.9500	-1.0218	8.4698	0.1812	-0.4491	-0.1317

CESSNA 182

MASS AND GEOMETRICAL CHARACTERISTICS

WEIGHT (LBS)	MOMENT OF INERTIA (SLUGS SQ.FT) IXX	MOMENT OF INERTIA (SLUGS SQ.FT) IZZ	SPAN (FT)	WING AREA (SQ.FT)
2650.	948.0	1967.0	36.08	174.38

FLIGHT CONDITIONS

VELOCITY (MPH)	DENSITY (SLUGS/CU.FT)	LIFT COEFFICIENT	ANGLE OF ATTACK(DEG)	FLIGHT PATH ANGLE(DEG)
150.0	0.002049	0.306	0.36	0.00

STABILITY DERIVATIVES

SIDESLIP	ROLL RATE	YAW RATE	CONTROL
CYB=-0.3191	CYP= 0.0011	CYR= 0.1697	CYC= 0.0000
CLB=-0.0798	CLP=-0.4628	CLR= 0.0844	CLC= 0.0000
CNB= 0.0550	CNP=-0.0337	CNR=-0.0864	CNC= 0.0000

INITIAL CONDITIONS

ANGLES(DEG)		RATES(DEG/SEC)	
ROLL	0.0	ROLL	0.0
YAW	0.0	YAW	0.0
SIDESLIP	10.0		

VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL

AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
0.0000	0.0000	0.0000	10.0000	0.0000	0.0000
0.0500	-0.2663	0.1093	9.8151	-9.4811	4.3509
0.1000	-0.8675	0.4310	9.4181	-13.9096	8.4669
0.1500	-1.6111	0.9495	8.8244	-15.4563	12.1985
0.2000	-2.3860	1.6427	8.0567	-15.3147	15.4444
0.2500	-3.1262	2.4847	7.1422	-14.1569	18.1372
0.3000	-3.7916	3.4465	6.1118	-12.3757	20.2352
0.3500	-4.3576	4.4979	4.9941	-10.2146	21.7185
0.4000	-4.8095	5.6081	3.8241	-7.8363	22.5855
0.4500	-5.1396	6.7464	2.6322	-5.3582	22.8511
0.5000	-5.3452	7.8836	1.4486	-2.8715	22.5445
0.5500	-5.4278	8.9920	0.3014	-0.4504	21.7077
0.6000	-5.3924	10.0464	-0.7837	1.8431	20.3926
0.6500	-5.2465	11.0243	-1.7843	3.9574	18.6603
0.7000	-5.0003	11.9066	-2.6812	5.8498	16.5779
0.7500	-4.6658	12.6775	-3.4583	7.4870	14.2172
0.8000	-4.2563	13.3249	-4.1057	8.8440	11.6523
0.8500	-3.7864	13.8405	-4.6141	9.9047	8.9579
0.9000	-3.2709	14.2197	-4.9800	10.6611	6.2071
0.9500	-2.7253	14.4615	-5.2031	11.1133	3.4702
1.0000	-2.1646	14.5681	-5.2869	11.2687	0.8126
1.0500	-1.6032	14.5451	-5.2376	11.1416	-1.7059
1.1000	-1.0548	14.4007	-5.0646	10.7520	-4.0328
1.1500	-0.5319	14.1457	-4.7795	10.1255	-6.1232
1.2000	-0.0457	13.7929	-4.3961	9.2916	-7.9407
1.2500	0.3943	13.3567	-3.9296	8.2832	-9.4578
1.3000	0.7803	12.8525	-3.3965	7.1352	-10.6560
1.3500	1.1061	12.2966	-2.8136	5.8840	-11.5256
1.4000	1.3676	11.7054	-2.1982	4.5662	-12.0653
1.4500	1.5622	11.0954	-1.5672	3.2178	-12.2821

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VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
1.5000	1.6994	10.4824	-0.9367	1.8734	-12.1904
1.5500	1.7502	9.8812	-0.3221	0.5654	-11.8111
1.6000	1.7471	9.3056	0.2627	-0.6768	-11.1709
1.6500	1.6840	8.7679	0.8053	-1.8269	-10.3011
1.7000	1.5663	8.2788	1.2951	-2.8627	-9.2368
1.7500	1.4000	7.8469	1.7234	-3.7660	-8.0156
1.8000	1.1921	7.4792	2.0837	-4.5229	-6.6766
1.8500	0.9503	7.1805	2.3711	-5.1239	-5.2595
1.9000	0.6824	6.9539	2.5833	-5.5640	-3.8033
1.9500	0.3966	6.8002	2.7197	-5.8421	-2.3459
2.0000	0.1009	6.7187	2.7814	-5.9615	-0.9226
2.0500	-0.1970	6.7068	2.7717	-5.9288	0.4342
2.1000	-0.4896	6.7605	2.6949	-5.7541	1.6955
2.1500	-0.7702	6.8744	2.5572	-5.4501	2.8366
2.2000	-1.0327	7.0418	2.3654	-5.0323	3.8370
2.2500	-1.2719	7.2555	2.1276	-4.5175	4.6811
2.3000	-1.4832	7.5072	1.8523	-3.9241	5.3579
2.3500	-1.6633	7.7884	1.5485	-3.2714	5.8610
2.4000	-1.8097	8.0903	1.2252	-2.5788	6.1889
2.4500	-1.9208	8.4044	0.8915	-1.8654	6.3442
2.5000	-1.9962	8.7220	0.5561	-1.1499	6.3336
2.5500	-2.0361	9.0351	0.2273	-0.4498	6.1675
2.6000	-2.0417	9.3364	-0.0874	0.2190	5.8593
2.6500	-2.0149	9.6190	-0.3812	0.8420	5.4252
2.7000	-1.9584	9.8771	-0.6482	1.4071	4.8832
2.7500	-1.8754	10.1058	-0.8836	1.9038	4.2532
2.8000	-1.7693	10.3013	-1.0836	2.3245	3.5557
2.8500	-1.6443	10.4606	-1.2454	2.6634	2.8117
2.9000	-1.5044	10.5820	-1.3676	2.9173	2.0422
2.9500	-1.3540	10.6647	-1.4495	3.0852	1.2673

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VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL

AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
3.0000	-1.1973	10.7090	-1.4915	3.1683	0.5062
3.0500	-1.0385	10.7159	-1.4951	3.1697	-0.2235
3.1000	-0.8816	10.6874	-1.4626	3.0943	-0.9061
3.1500	-0.7303	10.6263	-1.3969	2.9486	-1.5277
3.2000	-0.5878	10.5358	-1.3015	2.7404	-2.0771
3.2500	-0.4571	10.4199	-1.1807	2.4786	-2.5453
3.3000	-0.3487	10.2828	-1.0389	2.1727	-2.9258
3.3500	-0.2434	10.1289	-0.8808	1.8329	-3.2147
3.4000	-0.1578	9.9629	-0.7113	1.4695	-3.4107
3.4500	-0.0937	9.7894	-0.5351	1.0926	-3.5145
3.5000	-0.0486	9.6129	-0.3569	0.7123	-3.5293
3.5500	-0.0224	9.4378	-0.1812	0.3380	-3.4600
3.6000	-0.0145	9.2682	-0.0121	-0.0215	-3.3136
3.6500	-0.0241	9.1076	0.1468	-0.3586	-3.0982
3.7000	-0.0499	8.9594	0.2921	-0.6662	-2.8232
3.7500	-0.0902	8.8261	0.4212	-0.9389	-2.4989
3.8000	-0.1431	8.7101	0.5319	-1.1720	-2.1361
3.8500	-0.2067	8.6130	0.6228	-1.3624	-1.7461
3.9000	-0.2786	8.5358	0.6926	-1.5079	-1.3399
3.9500	-0.3567	8.4791	0.7412	-1.6078	-0.9283
4.0000	-0.4386	8.4429	0.7685	-1.6625	-0.5218
4.0500	-0.5222	8.4266	0.7752	-1.6734	-0.1297
4.1000	-0.6053	8.4295	0.7623	-1.6428	0.2392
4.1500	-0.6858	8.4501	0.7315	-1.5740	0.5775
4.2000	-0.7621	8.4866	0.6844	-1.4710	0.8787
4.2500	-0.8324	8.5372	0.6233	-1.3383	1.1378
4.3000	-0.8955	8.5996	0.5505	-1.1811	1.3511
4.3500	-0.9502	8.6715	0.4685	-1.0046	1.5162
4.4000	-0.9958	8.7504	0.3797	-0.8142	1.6320
4.4500	-1.0315	8.8339	0.2869	-0.6154	1.6986

****

VARIATION OF ROLL, YAW AND SIDESLIP ANGLES AND ROLL AND YAW RATES WITH TIME

TIME (SEC)	PHI (DEG)	PSI (DEG)	BETA (DEG)	ROLL VELOCITY (DEG/SEC)	YAW VELOCITY (DEG/SEC)
4.5000	-1.0572	8.9195	0.1923	-0.4136	1.7176
4.5500	-1.0729	9.0049	0.0986	-0.2138	1.6912
4.6000	-1.0787	9.0879	0.0078	-0.0208	1.6230
4.6500	-1.0752	9.1666	-0.0779	0.1613	1.5170
4.7000	-1.0629	9.2391	-0.1568	0.3285	1.3782
4.7500	-1.0426	9.3039	-0.2275	0.4778	1.2119
4.8000	-1.0154	9.3599	-0.2887	0.6067	1.0238
4.8500	-0.9823	9.4061	-0.3394	0.7132	0.8198
4.9000	-0.9445	9.4417	-0.3792	0.7962	0.6059
4.9500	-0.9031	9.4666	-0.4076	0.8549	0.3877
5.0000	-0.8594	9.4805	-0.4247	0.8896	0.1710
5.0500	-0.8145	9.4838	-0.4308	0.9008	-0.0393
5.1000	-0.7697	9.4768	-0.4264	0.8896	-0.2383
5.1500	-0.7259	9.4602	-0.4122	0.8577	-0.4218
5.2000	-0.6842	9.4349	-0.3892	0.8072	-0.5865
5.2500	-0.6455	9.4019	-0.3584	0.7403	-0.7294
5.3000	-0.6104	9.3624	-0.3211	0.6597	-0.8485
5.3500	-0.5797	9.3175	-0.2786	0.5682	-0.9421
5.4000	-0.5538	9.2686	-0.2322	0.4686	-1.0097
5.4500	-0.5329	9.2170	-0.1833	0.3640	-1.0512
5.5000	-0.5174	9.1639	-0.1332	0.2570	-1.0670
5.5500	-0.5072	9.1107	-0.0833	0.1505	-1.0585
5.6000	-0.5023	9.0584	-0.0346	0.0470	-1.0273
5.6500	-0.5024	9.0083	0.0116	-0.0512	-0.9755
5.7000	-0.5073	8.9612	0.0544	-0.1420	-0.9057
5.7500	-0.5165	8.9180	0.0930	-0.2237	-0.8205
5.8000	-0.5295	8.8793	0.1268	-0.2947	-0.7231
5.8500	-0.5458	8.8458	0.1551	-0.3541	-0.6165
5.9000	-0.5647	8.8178	0.1776	-0.4012	-0.5038
5.9500	-0.5857	8.7955	0.1941	-0.4354	-0.3883

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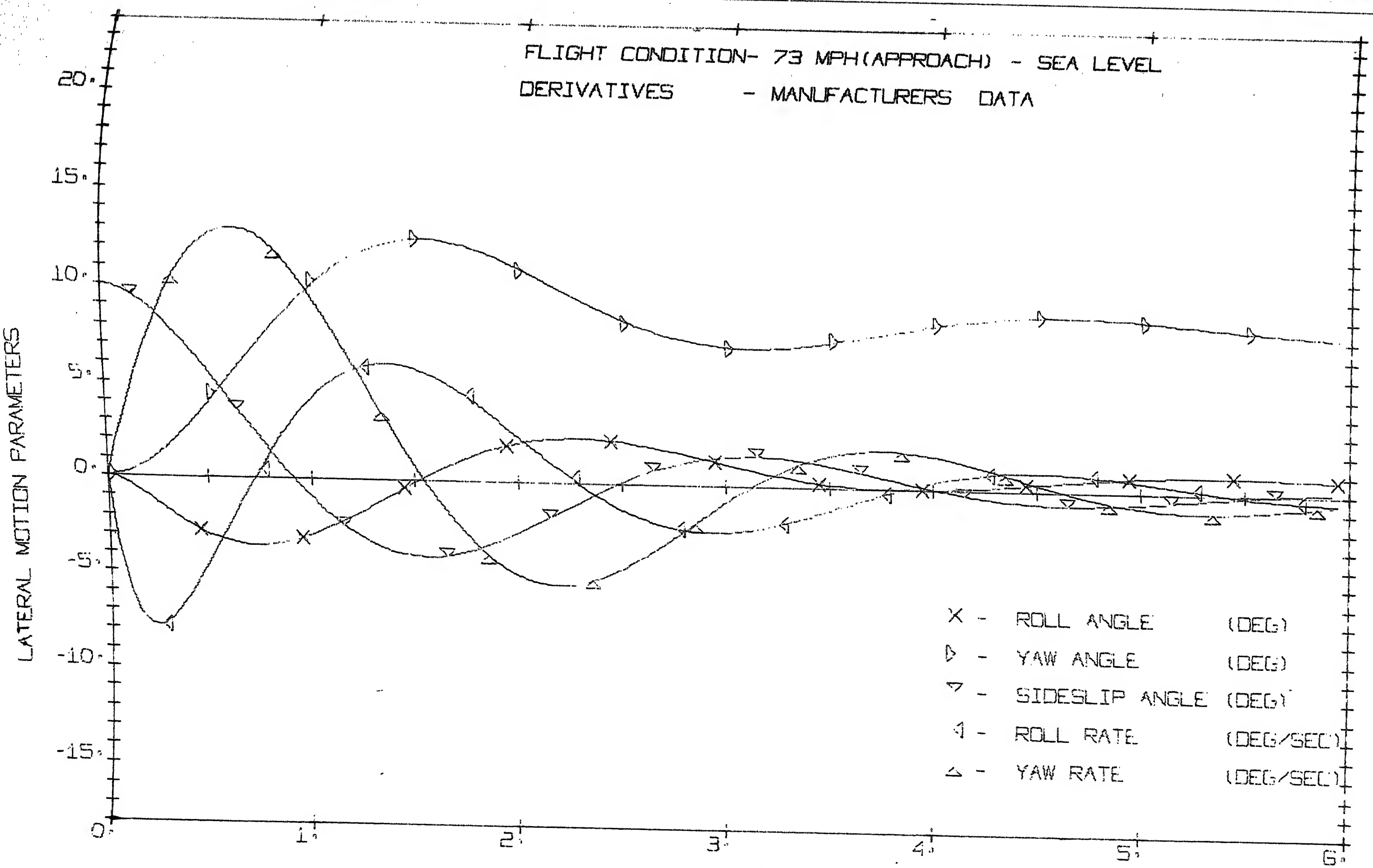
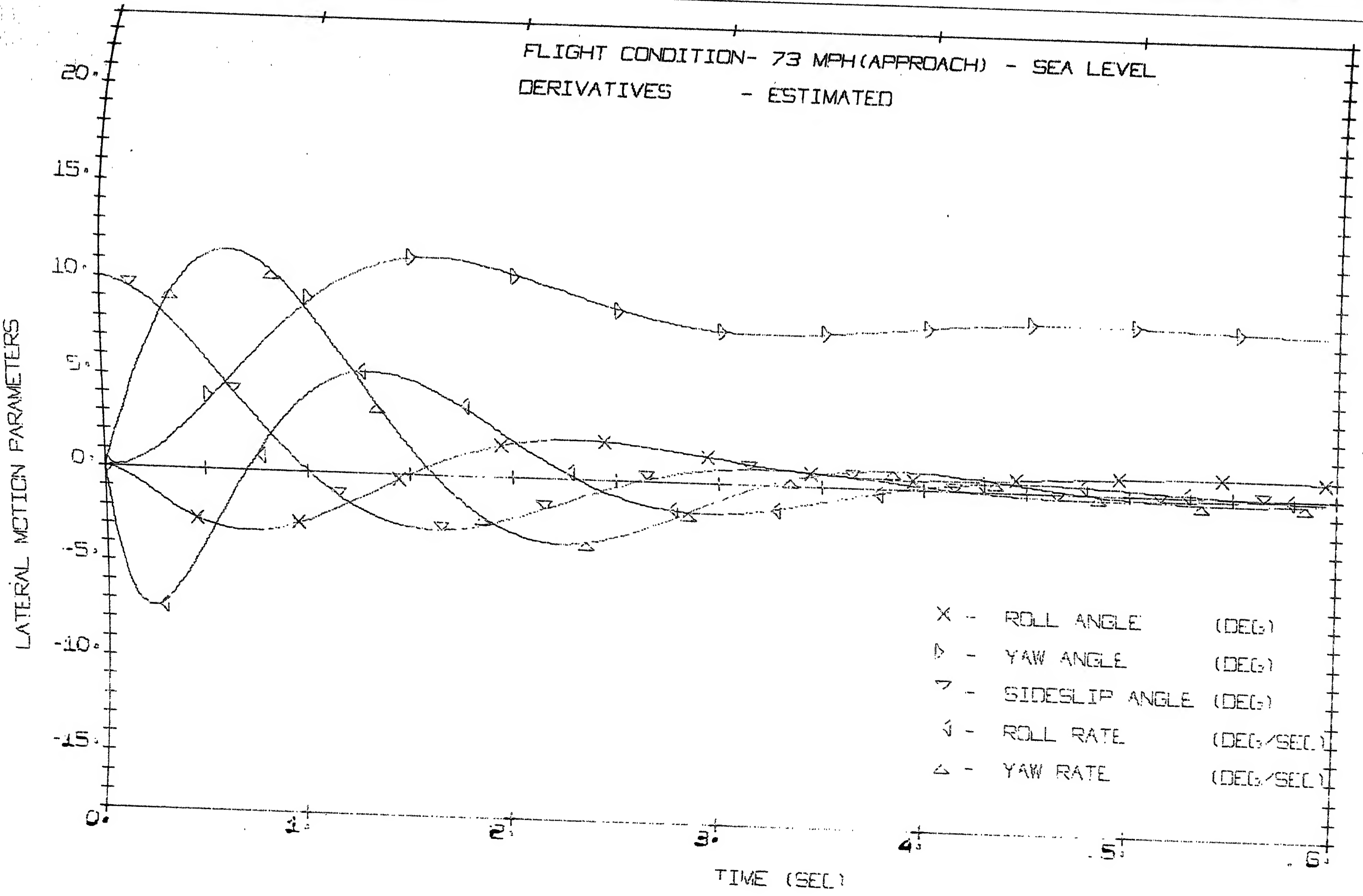
APPENDIX 'G'

PLOTS OF SOLUTIONS



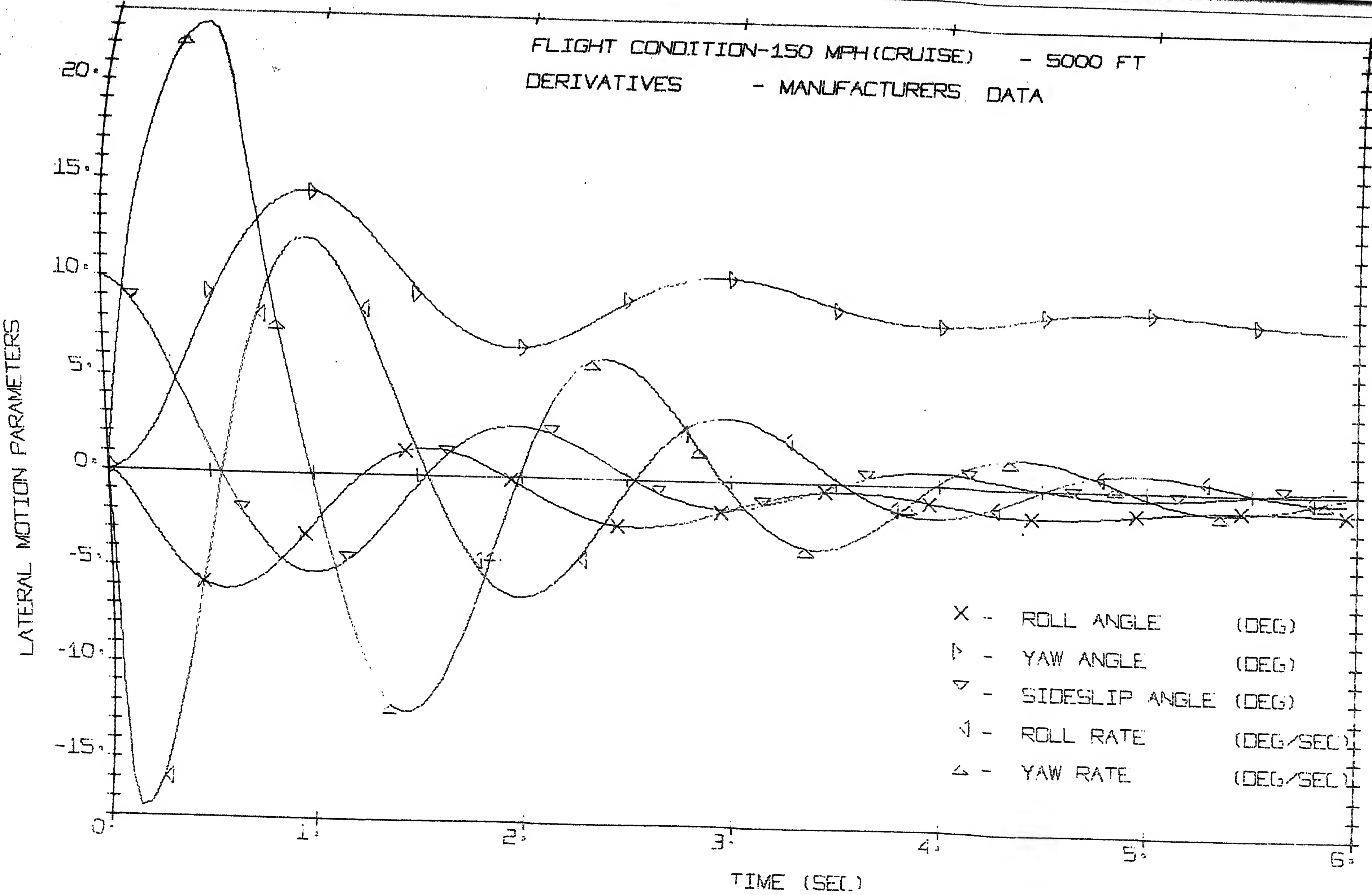
TIME HISTORY OF LATERAL MOTION PARAMETERS

(DISTURBANCE IN SIDESLIP ANGLE = 10 DEG)



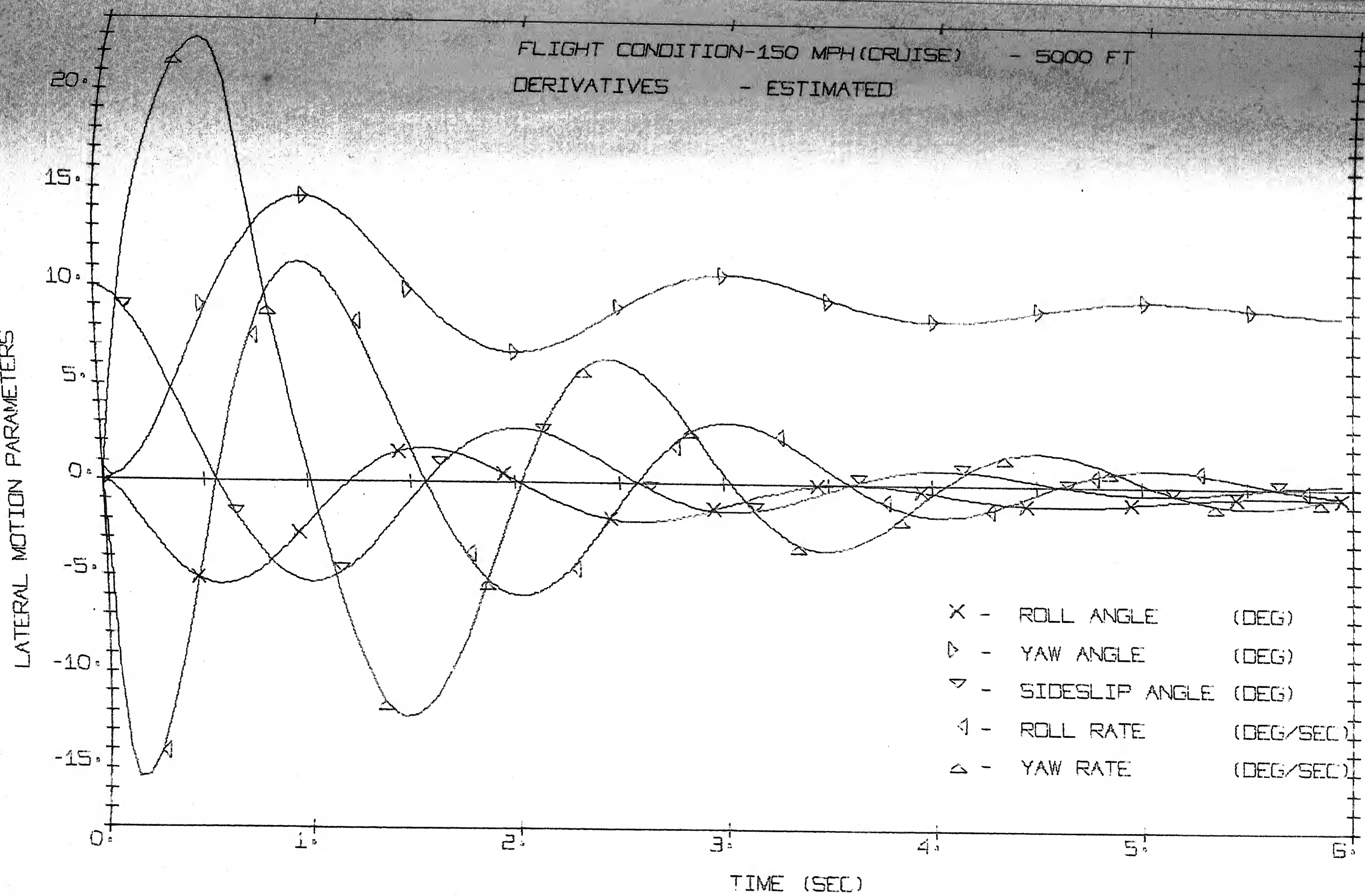
TIME HISTORY OF LATERAL MOTION PARAMETERS

(DISTURBANCE IN SIDESLIP ANGLE = 10 DEG)



TIME HISTORY OF LATERAL MOTION PARAMETERS

(DISTURBANCE IN SIDESLIP ANGLE = 10 DEG)



TIME HISTORY OF LATERAL MOTION PARAMETERS

(DISTURBANCE IN SIDESLIP ANGLE = 10 DEG)

TABLES

TABLE 1 : MASS CHARACTERISTICS OF CESSNA-182H

Parameter		Units
Weight	2650.0	lbs
I_{XX}	948.0	Slugs-ft^2.
I_{ZZ}	1967.0	"
ϵ	0.0	deg.

TABLE 2 : VARIATION OF FUSELAGE CROSS-SECTIONAL AREA ALONG BODY CENTRE LINE

Station (a)	Distance from Station Zero(in.) (b)	Cross-Section Area (Sq.Ft.) (c)	(b)x(c) (d)	Average Cross-sectional Area= $\frac{\Sigma(d)}{\Sigma(b)}$ (e)
0	0	0	0	
1	9.5	4.58	43.5	
2	39.0	13.78	537.2	
3	69.0	18.20	1266.0	
4	105.0	16.92	1779.0	8.3 Sq.ft.
5	132.0	13.00	1716.0	
6	150.0	9.44	1416.6	
7	213.0	4.33	922.2	
8	253.0	1.10	278.5	
	$\Sigma(b)=970.5$		$\Sigma(d)=7960.5$	

Average diameter $d_{av} = \sqrt{\frac{4x(e)}{\pi}} = 3.26$ ft.

TABLE 3 : GEOMETRIC CHARACTERISTICS AND OTHER RELATED CONSTANTS FOR CESSNA-182H

Component	Parameter	Value	Units	Remarks
Wing	Span b	36.083	ft.	
	Area S	174.38	Sq.ft.	
	Aspect Ratio A	7.46	-	
	Taper Ratio λ	0.672	-	
	Section C_{l_α}	0.105	(deg^{-1})	
	Sweep back angle Λ, $\Lambda_{c/2}$, $\Lambda_{c/4}$	0	deg.	
	M.A.C. $\bar{c}$	4.925	ft.	
	Position of C.G.	26.4% of M.A.C.	-	
	Dihedral Γ	1.75	deg.	
Horizontal Tail	Span b_H	11.667	ft.	
	Area S_H	41.3	Sq.ft.	
	Aspect Ratio A_H	3.33		
	Taper Ratio λ_H	0.59		
	C_{l_α} (Section)	0.105	(deg^{-1})	
	Λ; $\Lambda_{c/2}$, $\Lambda_{c/4}$	0	deg	
	$\bar{c}_H$	3.667	ft.	
	Γ_H	0	deg	

TABLE 3 : (CONTD.)

Component	Parameter	Value	Unit	Remarks
Vertical Tail	Span b_V	5.333	ft.	
	Area S_V	19.36	Sq.ft.	
	Aspect Ratio $A_V = \frac{b_V^2}{S_V}$	1.468	-	
	λ_V	0.526	-	
	C_{l_α}	0.105	(deg.^{-1})	
	Λ	45	deg.	
	$\Lambda_{c/4}$	40	deg.	
	$\Lambda_{c/2}$	36	deg.	
	$\bar{c}_V$	3.667	ft.	
	z_V	2.4166	ft.	
Body	z_W	-2.083	ft.	
	d	4.833	ft.	
	$z_W/\frac{d}{2}$	-0.862	-	
	l_B	24.125	ft.	
	X_1	165.0	in.	Fig.5
	X_1/l_B	0.57	-	
	X_o/l_B	0.69	-	Fig.6

TABLE 3 : (CONTD.)

Component	Parameter	Value	Unit	Remarks
Body	X_o	192.5	in.	
	S_o	6.0	Sq.ft.	Fig.5
	l_d	16.667	ft.	Hor.distance of L.E. of M.A.C. to M.A.C. of vertical tail.
	l_V	15.367	ft.	Fig.18
	X_m	6.667	ft.	
	S_{B_S}	68.0	Sq.ft.	
	h	4.833	ft.	
	$l_B/4$	6.033	ft.	
	$3l_B/4$	18.1	ft.	
	h_1	4.7083	ft.	
	h_2	2.0	ft.	
	$\sqrt{h_1/h_2}$	1.535	-	
	w	4.0833	ft.	
	h/w	1.182	-	
	l_B^2/S_{B_S}	8.55	-	
	Average cross-sectional area	8.30	Sq.ft.	Weighted average from Table 2.
	d_{av}	3.26	ft.	

TABLE 3: (CONTD.)

Component	Parameter	Value	Unit	Remarks
Wing-Body	l_f	7.58	ft.	
	l_f/b	0.204		
Tail-Body	$2r_1$	1.5	ft.	
	$b_V/2r_1$	3.55		
	X	1.667	ft.	
	$C_V = C_{R_V}$	4.75	ft.	Tail root chord is taken as the chord along the body centre line.
	X/C_V	0.351	-	
	Z_H	0.0	ft.	
	Z_H/b_V	0.0	-	
	l_{f_H}	20.5	ft.	
	l_{f_H}/b_H	1.76	-	
Miscellaneous	Mach No. M	0	-	
	$\beta = \sqrt{1-M^2}$	1	-	
	$A/\cos\Lambda_{c/4} = A$	2.135	-	
	$M \cos\Lambda_{c/2}$	0.0	-	
	S_H/S_V	2.135	-	

TABLE 3 : (CONTD.)

Component	Parameter	Value	Unit	Remarks
Other Factors	K_i	1.72	-	Fig.4
	k	1.0	-	Fig.7
	$\frac{A_{V(B)}}{A_V}$	1.2	-	Fig.8
	$\frac{A_{V(HB)}}{A_V(B)}$	1.0	-	Fig.9
	K_H	1.14	-	Fig.10
	$((\frac{C_{l_\beta}}{C_L})_{\Lambda_{c/2}})_W$	0.0	$(deg.^{-1})$	Fig.11
	$((\frac{C_{l_\beta}}{C_L})_{\Lambda_{H_{c/2}}})_H$	0.0	$(deg.^{-1})$	Fig.11
	K_{M_Λ}	1.0	-	Fig.12
	$(K_f)_W$	0.99	-	Fig.13
	$(K_f)_H$	1.00	-	Fig.13
	$((C_{l_\beta}/C_L)_A)_W$	-0.0005	$(deg.^{-1})$	Fig.14
	((" $)_A)_H$	-0.0021	(")	Fig.14
	$(C_{l_\beta}/\Gamma)_W$	-0.00023	$(deg.^{-2})$	Fig.15
	(" $)_H$	-0.000155	(")	Fig. 15

TABLE 3 : (CONTD.)

Component	Parameter	Value	Units	Remarks
	$(K_{M_\Gamma})_W$	1.00	-	Fig.16
	$(K_{M_\Gamma})_H$	1.00	-	Fig.16
	$(\Delta C_{l_\beta}/\theta \tan \Lambda_{c/4})_W$	-0.000037	(deg^{-2})	Fig.17
	(" " $)_H$	-0.000025	"	Fig.17
	K_N	.00115	-	Fig.19
	$\mathcal{K}$	0.9584	-	
	$(\beta A/\mathcal{K})_W$	7.79	-	
	$(\beta A/\mathcal{K})_H$	3.45	-	
	$\left(\frac{\beta\, C_{l_p}}{\mathcal{K}}\right)_W$	-0.485	(rad^{-1})	Fig.23
	(" $)_H$	-0.24	(")	Fig.23
	$\left(\frac{C_{n_p}}{C_L}\right)_{\substack{C_L=0 \\ M=0}}$	-0.1085	(")	
	$(\ " \ )_{\substack{C_L=0 \\ M}}$	-0.1085	(")	
	$(\Delta C_{n_p}/\theta)$	0.000057	$\frac{1}{(rad\text{-}deg)}$	Fig.25

TABLE 3: (CONTD.)

Component	Parameter	Value	Units	Remarks
C_{l_r}	$(C_{l_r}/C_L)_{C_L=0, M=0}$	0.24	(rad^{-1})	Fig.26
	$(\Delta C_{l_r}/\theta)$	-0.0145	$\frac{1}{(rad\text{-}deg)}$	Fig.27
	(C_{n_r}/C_L^2)	-0.02	(rad^{-1})	Fig.28
	$(C_{n_r}/\bar{C}_{D_o})$	-0.3	"	Fig.29
	C_{L_α}	4.69	"	
	$C_{L_{\alpha_W}}$	4.65	"	
	$C_{L_{\alpha_H}}$	3.475	"	
	K_β	0.14		Fig.21
	$\frac{(\Delta C_{n_p})_1}{C_L}$	-0.06	(rad^{-1})	Fig.24
	$\frac{(\Delta C_{n_p})_2}{(C_{D_o})_\alpha}$	9.5	(deg^{-1})	Fig.24

TABLE 4 : FLIGHT CONDITIONS AND ALLIED PARAMETERS

Parameters / Flight Conditions	Velocity MPH	Altitude Ft.	Density Slugs/cu.ft.
Approach	73	S.L.	.0002378
Cruising	150	5,000	.0002049

TABLE 5 : PARAMETERS DEPENDENT ON FLIGHT CONDITIONS

Parameters	Unit	Approach	Cruising
C_L	-	1.1150	0.306
C_D	-	0.132	0.032
C_{D_α}	-	0.547	0.121
C_{L_H}	-	0.3975	0.01393
ν	ft.^{2}/sec.	156.4×10^{-6}	176.6×10^{-6}
$R_e = \frac{l_B V}{\nu}$	-	16.51×10^6	30.15×10^6
K_{R_1}	-	1.61	1.74

TABLE 6(a) : COMPARISON OF CONTRIBUTION OF VARIOUS COMPONENTS TO STABILITY DERIVATIVES ESTIMATED FROM REFERENCES 1 TO 5 (APPROACH AT 73 mph)

Derivative	Ref.1	Ref.2	Ref.3	Ref.4	Ref.5	Manufacturers Data
$C_{Y_{\beta_W}}$	N/A	N/A	N/A	N/A	-0.0099	N/A
$C_{Y_{\beta_B}}$	N/A	N/A	N/A	N/A	-0.1186	N/A
$C_{Y_{\beta_V}}$	N/A	N/A	N/A	N/A	-0.1908	N/A
C_{Y_β}	N/A	N/A	N/A	N/A	-0.3191	-0.303
$C_{l_{\beta_{WB}}}$	-0.1208	-0.1208	-0.1817	-0.01208	-0.0919	N/A
$C_{l_{\beta_H}}$	Neg.	-0.0028	Neg.	Neg.	-0.0024	N/A
$C_{l_{\beta_V}}$	0.0026	0.0024	0.0026	0.0024	0.0024	N/A
C_{l_β}	-0.1182	-0.1212	-0.1791	-0.1183	-0.0919	-0.097
$C_{n_{\beta_W}}$	0.0249	0.0274	Neg.	0.0249	Neg.	N/A
$C_{n_{\beta_B}}$	-0.0492	-0.0492	-0.0012	-0.0492	-0.0276	N/A
$C_{n_{\beta_V}}$	0.0928	0.0928	0.0928	0.0928	0.0857	N/A
C_{n_β}	0.0684	0.0709	0.0916	0.0684	0.0580	0.0701

TABLE 6(a) : (CONTD.)

Derivative	Ref.1	Ref.2	Ref.3	Ref.4	Ref.5	Manufacturers Data
C_{Y_p}	0.0353	Neg.	Neg.	0.0304	0.0049	-0.2134
$C_{l_{p_W}}$	-0.4596	-0.4596	-0.4596	-0.4732	-0.4596	N/A
$C_{l_{p_H}}$	Neg.	Neg.	Neg.	-0.0033	-0.0033	N/A
$C_{l_{p_V}}$	N/A	N/A	N/A	-0.0004	-0.0017	N/A
C_{l_p}	-0.4596	-0.4596	-0.4596	-0.4770	-0.4646	-0.494
$C_{n_{p_W}}$	-0.0508	-0.0508	-0.1394	-0.0508	-0.1216	N/A
$C_{n_{p_V}}$	-0.0159	-0.0137	Neg.	-0.0137	-0.0022	N/A
C_{n_p}	-0.0667	-0.0645	-0.1394	-0.0645	-0.1238	-0.096
C_{Y_r}	0.2182	Neg.	Neg.	0.1714	0.1714	0.2010
$C_{l_{r_W}}$	0.2676	0.2676	0.2787	0.2676	0.2676	N/A
$C_{l_{r_H}}$	Neg.	0.0006	Neg.	Neg.	Neg.	N/A
$C_{l_{r_V}}$	-0.0028	-0.0022	Neg.	-0.0022	-0.0022	N/A
C_{l_r}	0.2648	0.2660	0.2787	0.2654	0.2654	0.2039
$C_{n_{r_W}}$	-0.0484	-0.0484	-0.0484	-0.0484	-0.0484	N/A

TABLE 6(a):(CONTD.)

Derivative	Ref.1	Ref.2	Ref.3	Ref.4	Ref.5	Manufacturers Data
$C_{n_{r_V}}$	-0.077	-0.077	-0.077	-0.077	-0.077	N/A
C_{n_r}	-0.1254	-0.1254	-0.1254	-0.1254	-0.1254	-0.115

Note: 1. N.A. - No method available or value not available.

Neg. - Neglected.

Note: 2. The values of roll-rate derivatives estimated by the method of Ref.6 are as follows:

$C_{y_p} \doteq -0.1132$ $\qquad C_{l_p} = -0.4783$ $\qquad C_{n_p} = -0.097$

TABLE 6(b) : COMPARISON OF CONTRIBUTION OF VARIOUS COMPONENTS TO STABILITY DERIVATIVES ESTIMATED FROM REFERENCES 1 TO 5 (CRUISE AT 150 m.p.h)

Derivative	Ref.1	Ref.2	Ref.3	Ref.4	Ref.5	Manufacturer's Data
$C_{Y_{\beta_W}}$	N/A	N/A	N/A	N/A	-0.0099	N/A
$C_{Y_{\beta_B}}$	N/A	N/A	N/A	N/A	-0.1184	N/A
$C_{Y_{\beta_V}}$	N/A	N/A	N/A	N/A	-0.1908	N/A
C_{Y_β}	N/A	N/A	N/A	N/A	-0.3191	-0.303
$C_{l_{\beta_{WB}}}$	-0.0884	-0.0884	-0.0675	-0.0884	-0.0688	N/A
$C_{l_{\beta_H}}$	Neg.	0.0013	Neg.	Neg.	0.0013	N/A
$C_{l_{\beta_V}}$	-0.0133	-0.0123	-0.0133	-0.0123	-0.0123	N/A
C_{l_β}	-0.1017	-0.1006	-0.0808	-0.1007	-0.0798	-0.0923
$C_{n_{\beta_W}}$	0.0019	0.0026	Neg.	0.0019	Neg.	N/A
$C_{n_{\beta_B}}$	-0.0492	-0.0492	-0.0012	-0.0492	-0.0299	N/A
$C_{n_{\beta_V}}$	0.0918	0.0918	0.0918	0.0918	0.0849	N/A
C_{n_β}	0.0445	0.0452	0.0907	0.0445	0.0550	0.0587

TABLE 6(b) : (CONTD.)

Derivative	Ref.1	Ref.2	Ref.3	Ref.4	Ref.5	Manufacturer's Data
C_{Y_p}	0.0319	Neg.	Neg.	0.00105	-0.0245	-0.0752
$C_{l_{p_W}}$	-0.4596	-0.4596	-0.4596	-0.461	-0.4596	N/A
$C_{l_{p_H}}$	Neg.	Neg.	Neg.	-0.0033	-0.0033	N/A
$C_{l_{p_V}}$	N/A	N/A	N/A	0.0001	-0.0017	N/A
C_{l_p}	-0.4596	-0.4596	-0.4596	-0.4643	-0.4646	-0.484
$C_{n_{p_W}}$	-0.0184	-0.0184	-0.0383	-0.0184	-0.0333	N/A
$C_{n_{p_V}}$	-0.0142	-0.0005	Neg.	-0.0005	0.0109	N/A
C_{n_p}	-0.0326	-0.0188	-0.0383	-0.0188	-0.0224	-0.0278
C_{Y_r}	0.2160	Neg.	Neg.	0.1697	0.1697	0.214
$C_{l_{r_W}}$	0.0736	0.0736	0.0766	0.0736	0.0736	N/A
$C_{l_{r_H}}$	Neg.	-0.0003	Neg.	Neg.	Neg.	N/A
$C_{l_{r_V}}$	0.0139	0.0109	Neg.	0.0109	0.0109	N/A
C_{l_r}	0.0874	0.0841	0.0766	0.0845	0.0845	0.0798
$C_{n_{r_W}}$	-0.0109	-0.0109	-0.0109	-0.0109	-0.0109	N/A

TABLE 6(b) : (CONTD.)

Derivative	Ref.1	Ref.2	Ref.3	Ref.4	Ref.5	Manufacturer's Data
$C_{n_{r_V}}$	=0.0755	-0.0755	-0.0755	-0.0755	-0.0755	N/A
C_{n_r}	-0.0864	-0.0864	-0.0864	-0.0864	-0.0864	-0.0937

Note 1 : N.A. : No method available or value not available.

Neg. : Neglected.

Note 2 : The values of roll-rate derivatives estimated by the method of Ref.6 are as follows:

$C_{Y_p} = -0.0403$ $\quad C_{l_p} = -0.4661$ $\quad C_{n_p} = -0.032$

TABLE 7(a) : COMPARISON OF ESTIMATED AND AVAILABLE VALUES OF STABILITY DERIVATIVES (APPROACH AT 73 m.p.h.)

Derivative	Percentage Error From the Manufacturer's Values				
	Ref.1	Ref.2	Ref.3	Ref.4	Ref.5
C_{l_β}	21.8*	22.0*	84.6*	22.0*	5.3**
C_{n_β}	2.4*	1.2**	30.6**	2.4*	17.2*
C_{Y_β}	N.E.	N.E.	N.E.	N.E.	5.3*
C_{l_p}	6.0**	6.0**	6.0**	3.5**	5.9**
C_{n_p}	30.5**	32.8**	45.2**	32.8**	28.9*
C_{Y_p}	116.0**	N.E.	N.E.	114.0**	102.0**
C_{l_r}	29.8**	30.5**	36.7**	30.1**	30.1**
C_{n_r}	9.0*	9.0*	9.0*	9.0*	9.0*
C_{Y_r}	8.6**	N.E.	N.E.	14.7*	14.7*

Note: N.E. - not estimated

* means less than the manufacturer's value

** means more than the manufacturer's value

TABLE 7(b) : COMPARISON OF ESTIMATED AND AVAILABLE VALUES OF STABILITY DERIVATIVES (CRUISE AT 150 m.p.h.)

Derivative	Percentage Error From the Manufacturer's Values				
	Ref.1	Ref.2	Ref.3	Ref.4	Ref.5
C_{l_β}	10.2*	9.1*	12.5**	9.1*	13.6**
C_{n_β}	24.2*	23.0*	54.5**	24.2*	6.4*
C_{Y_β}	N.E.	N.E.	N.E.	N.E.	5.3*
C_{l_p}	6.0**	6.0**	6.0**	4.0**	4.0**
C_{n_p}	17.3**	32.2*	37.8**	32.2*	19.5*
C_{Y_p}	142.5**	N.E.	N.E.	101.4**	67.4**
C_{l_r}	9.6**	5.4**	3.9*	5.8**	5.8**
C_{n_r}	7.8**	7.8**	7.8**	7.8**	7.8**
C_{Y_r}	1.0**	N.E.	N.E.	20.7*	20.7**

Note: N.E. means not estimated.

* means less than manufacturer's value.

** means more than manufacturer's value.

TABLE 8 : A CONSOLIDATED LIST OF ESTIMATED AND AVAILABLE VALUES OF STABILITY DERIVATIVES

Derivatives \ Flight condition	Cruise		Approach	
	Estimated	Manufacturer's Data	Estimated	Manufacturer's Data
C_{l_β}	-0.0798	-0.0923	-0.0919	-0.097
C_{n_β}	0.0550	0.0587	-0.0580	0.0701
C_{Y_β}	-0.3191	-0.303	-0.3191	-0.303
C_{l_p}	-0.4628	-0.484	-0.4633	-0.494
C_{n_p}	-0.0337	-0.0278	-0.1353	-0.0960
C_{Y_p}	0.0010	-0.0752	0.0304	-0.2134
C_{l_r}	0.0845	0.0798	0.2654	0.2039
C_{n_r}	-0.0864	-0.0937	-0.1254	-0.1150
C_{Y_r}	0.1697	0.214	0.1714	0.201

TABLE 9(a) : PERIOD AND DAMPING CHARACTERISTICS

(APPROACH AT 73 m.p.h)

Mode	Root		$T_{1/2}$		P	
	Estimated	Manufacturer's Data	Estimated	Manufacturer's Data	Estimated	Manufacturer's Data
Spiral	-.0101	-.024	126.7	53.5	-	-
Rolling	-12.31	-13.56	0.1044	0.0945	-	-
Dutch Roll	-1.408 ±i 3.64	-1.08 ±i 3.74	0.9119	1.189	3.197	3.112

TABLE 9(b) : PERIOD AND DAMPING CHARACTERISTICS
(CRUISE AT 150 m.p.h)

Mode	Root		$T_{1/2}$		P	
	Est.	Mfr. Data	Est.	Mfr. Data	Est.	Mfr. Data
Spiral	-.0117	-.01887	61.78	38.45	-	-
Rolling	-13.09	-13.712	0.0554	0.0529	-	-
Dutch Roll	- 0.65 ± i 3.25	- 0.688 ±i3.325	1.1066	1.0539	2.023	1.978

$$T_{1/2} = -\frac{0.69}{Re(\lambda)} \times \tau \quad ; \quad P = \frac{2\pi}{Im(\lambda)} \times \tau$$

FIGURES

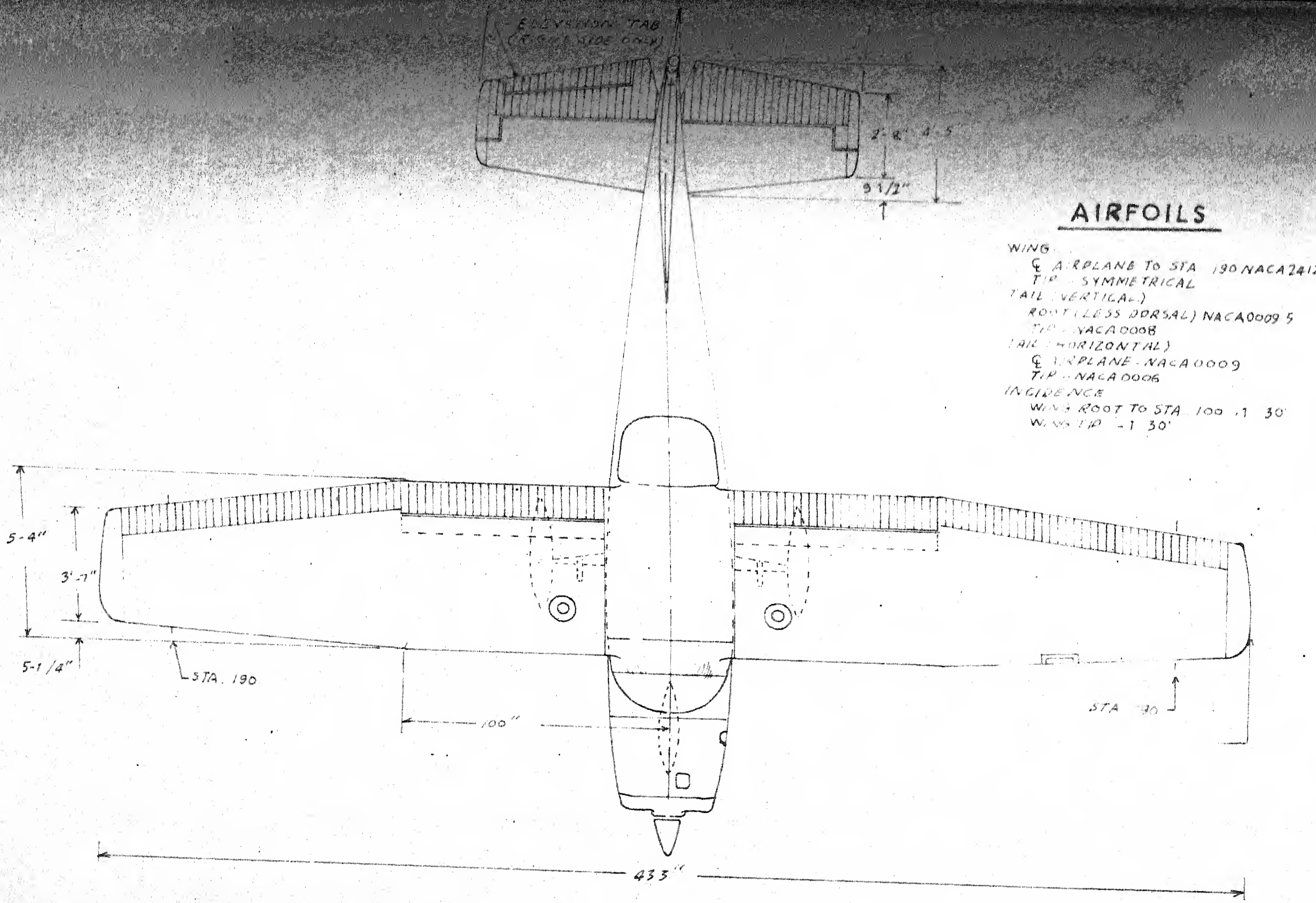


Fig. 1(a) Cessna 182-H Airplane

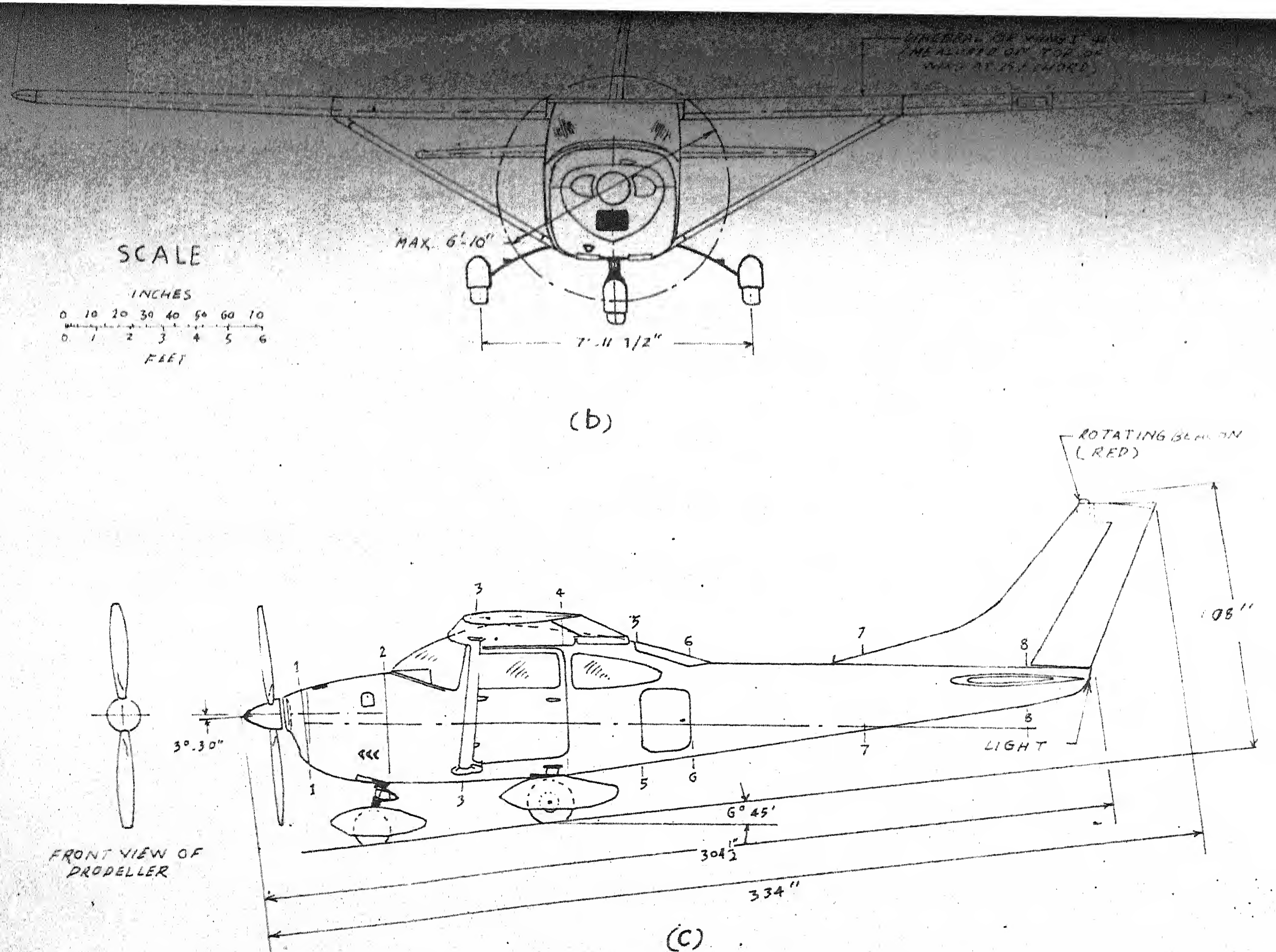


Fig-1(b&C)_ Cessna 182-H Airplane

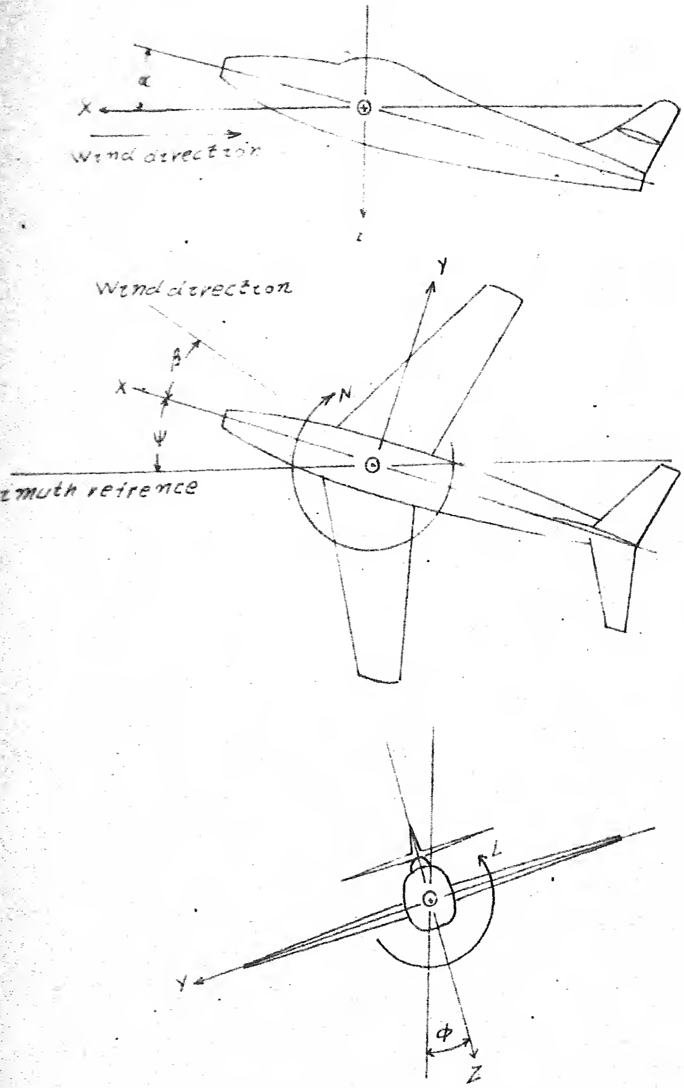


Fig. 2 - System of Axes.

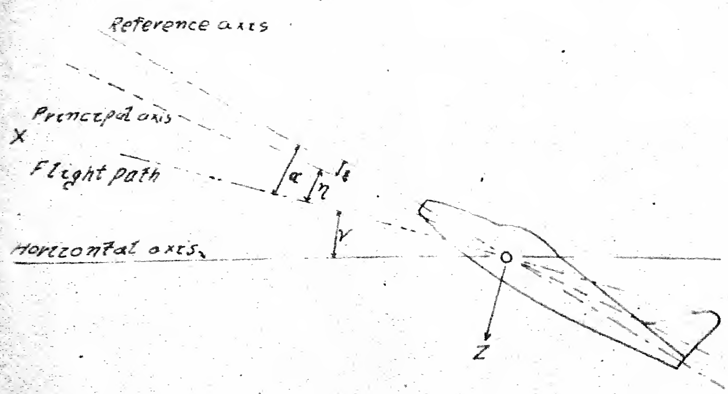


Fig. 3. System of Axes and Angular Relationships in Flight.

ALL SPEEDS

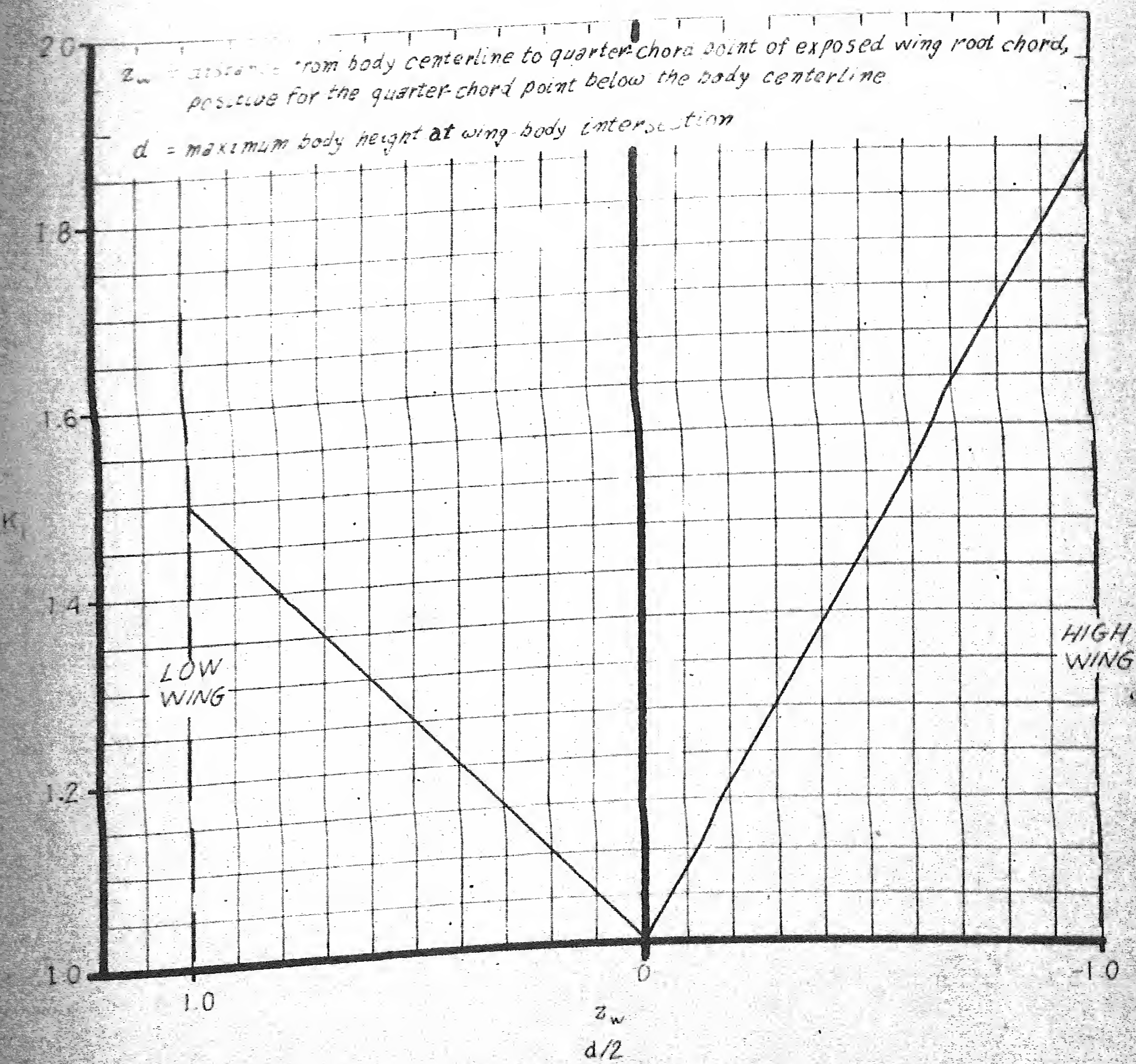


Figure 4. Wing-Body Interference Factor for Wing-Body Sideslip Derivative C_{y_β}

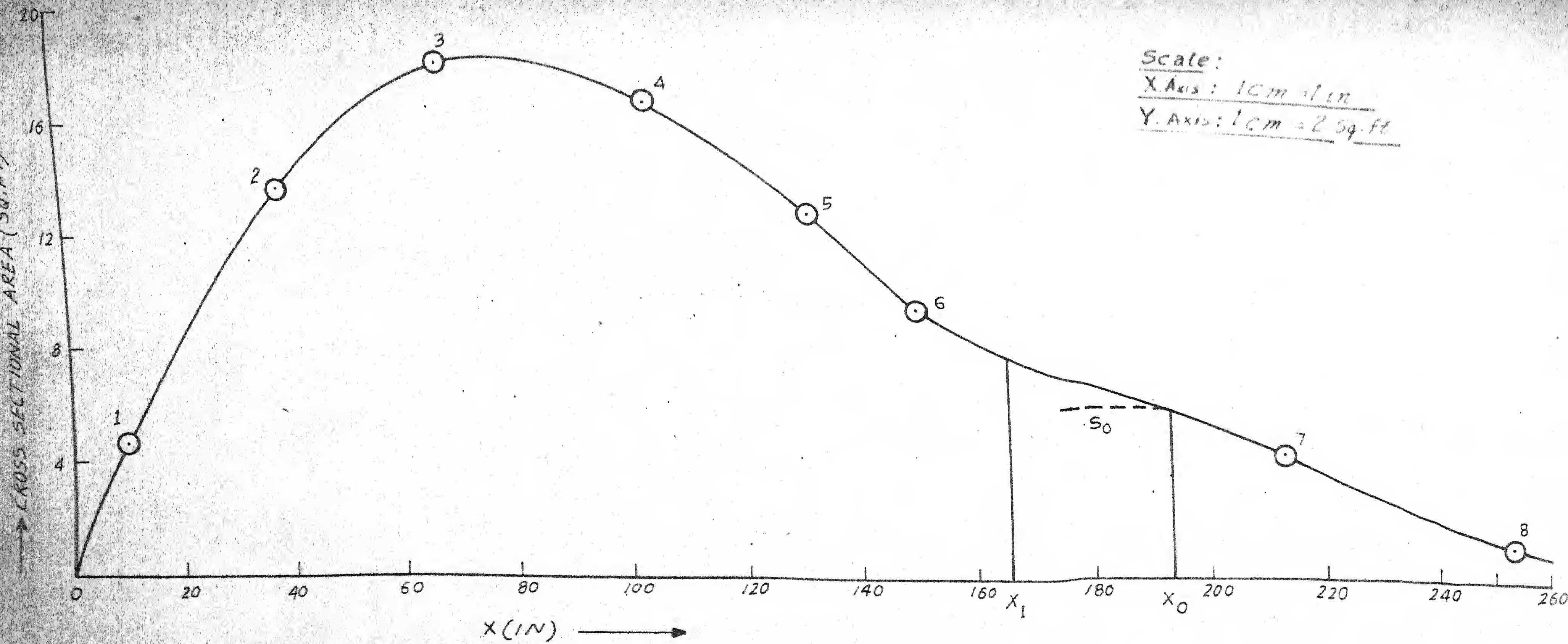


Fig. 5. Variation of Fuselage Cross-Sectional Area Along Body Centre line.

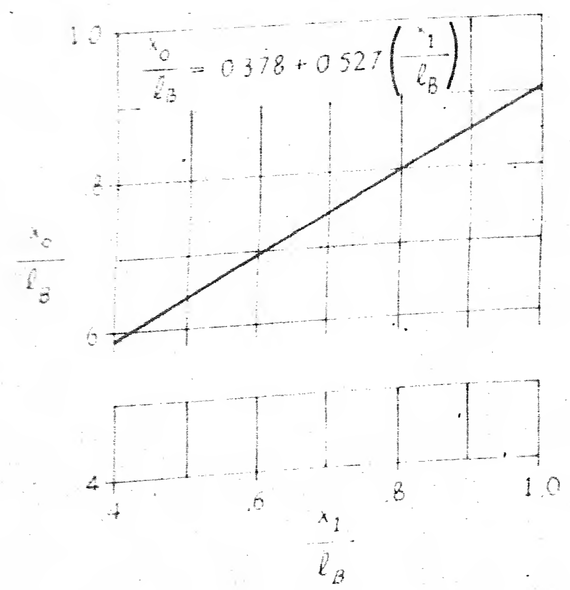
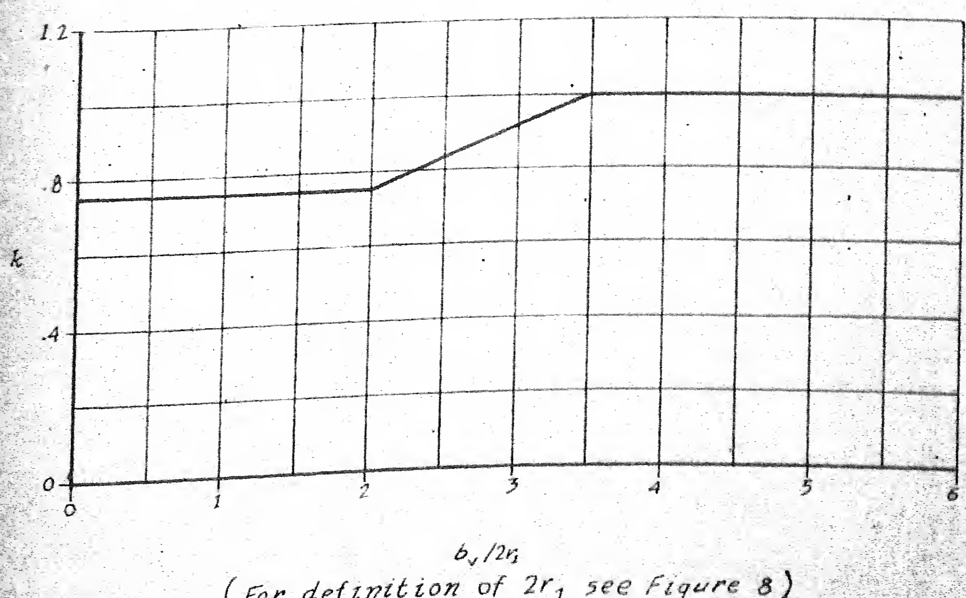


Figure 6. Body Station Where Flow Becomes Viscous



(For definition of $2r_1$ see Figure 8)

Figure 7. Empirical Factor for Estimating the Sideslip Derivative $C_{y_{\beta_V}}$ for Single Vertical Tails

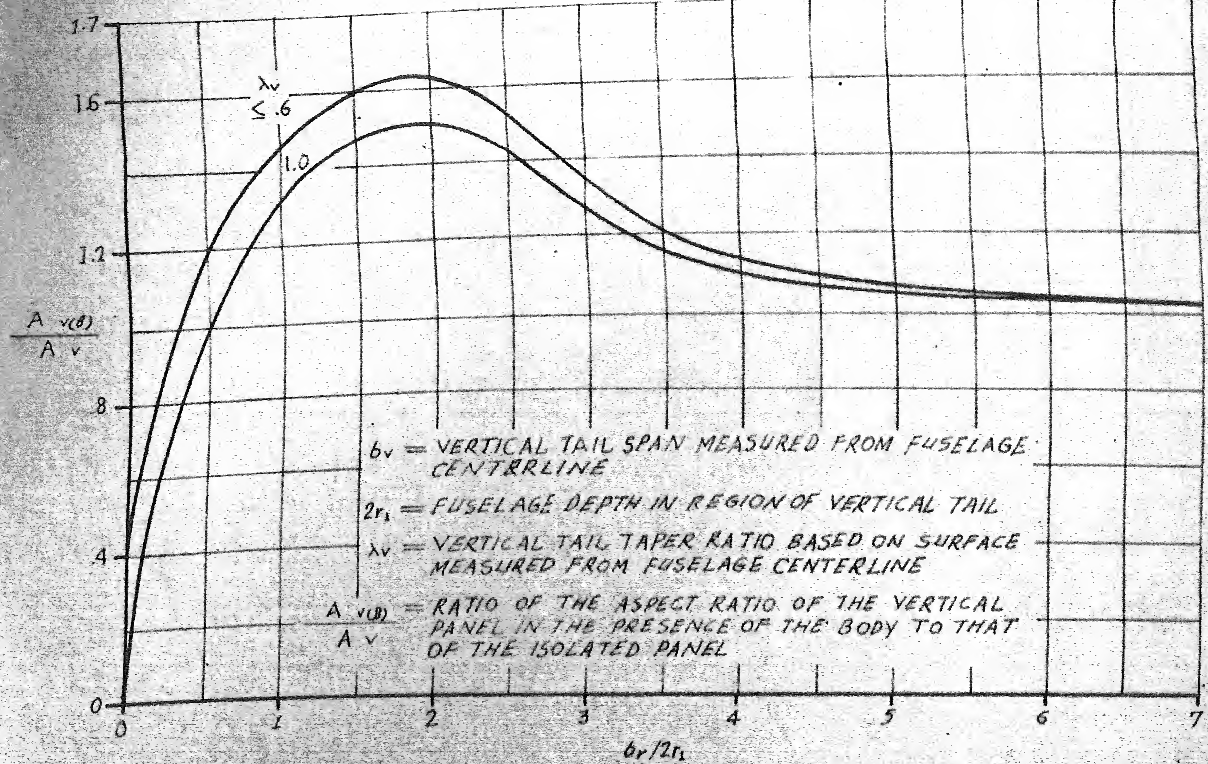


Figure 8. Aspect Ratio Factor due to Body Interference, used for Estimating the Sideslip Derivative $C_{y_{\beta_V}}$ for Single Vertical Tails

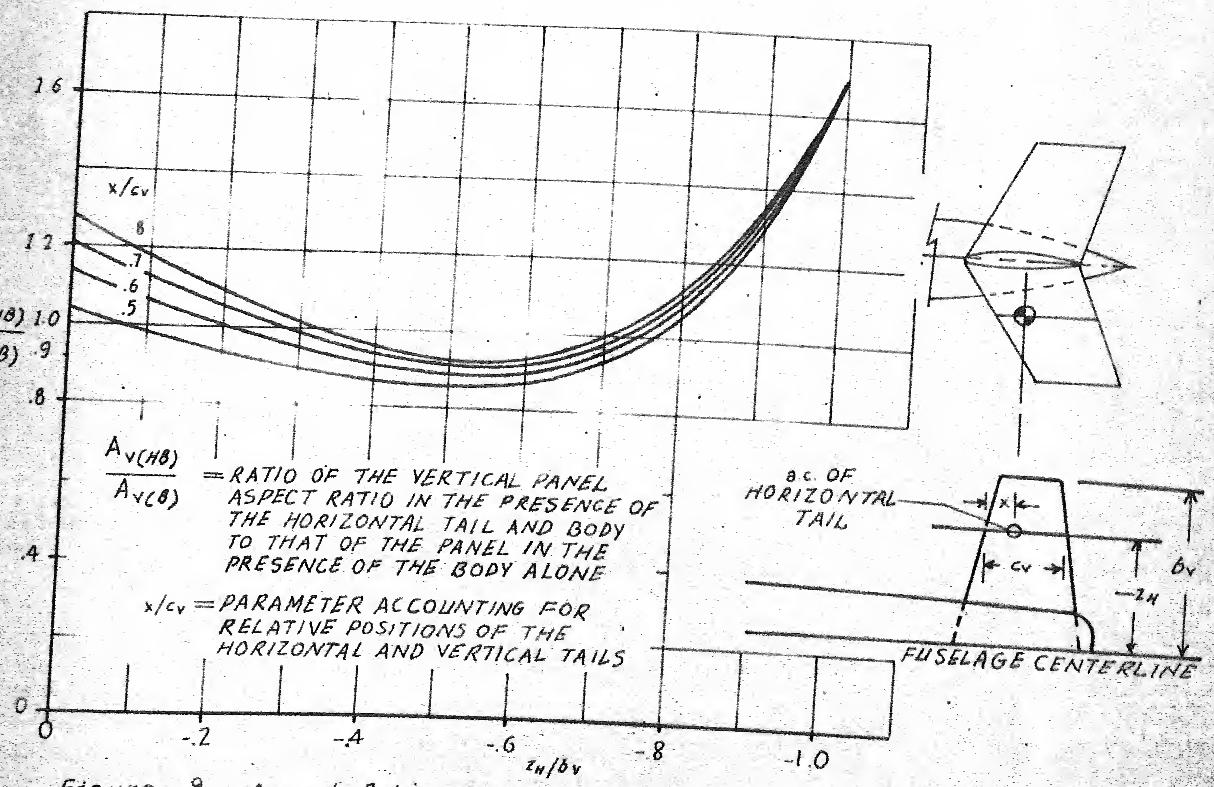


Figure 9. Aspect Ratio Factor due to Horizontal Tail Interference, Used for Estimating the Sideslip Derivative $C_{Y_{\beta_V}}$ for Single Vertical Tails

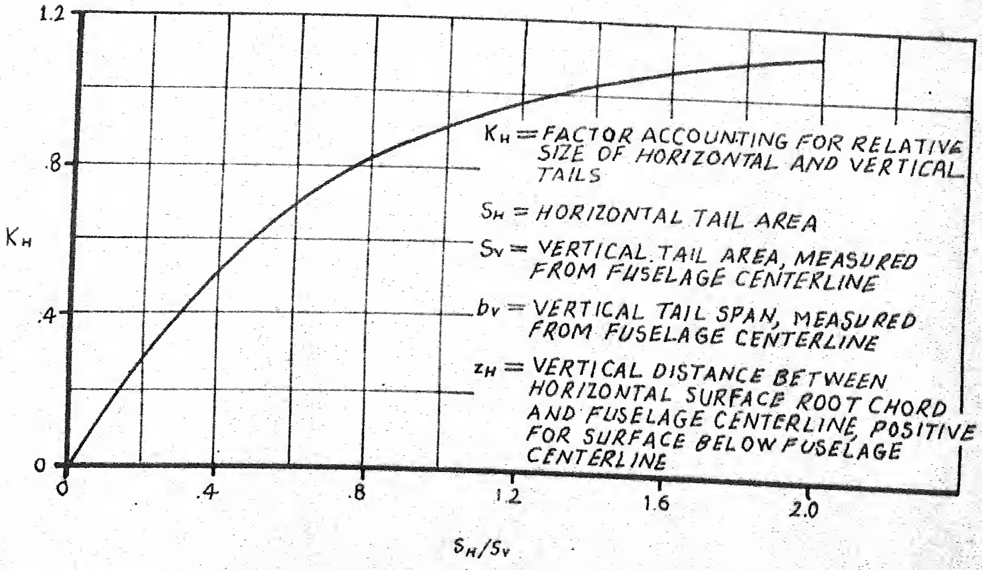


Figure 10. Factor Accounting for Relative Size of Horizontal and Vertical Tails

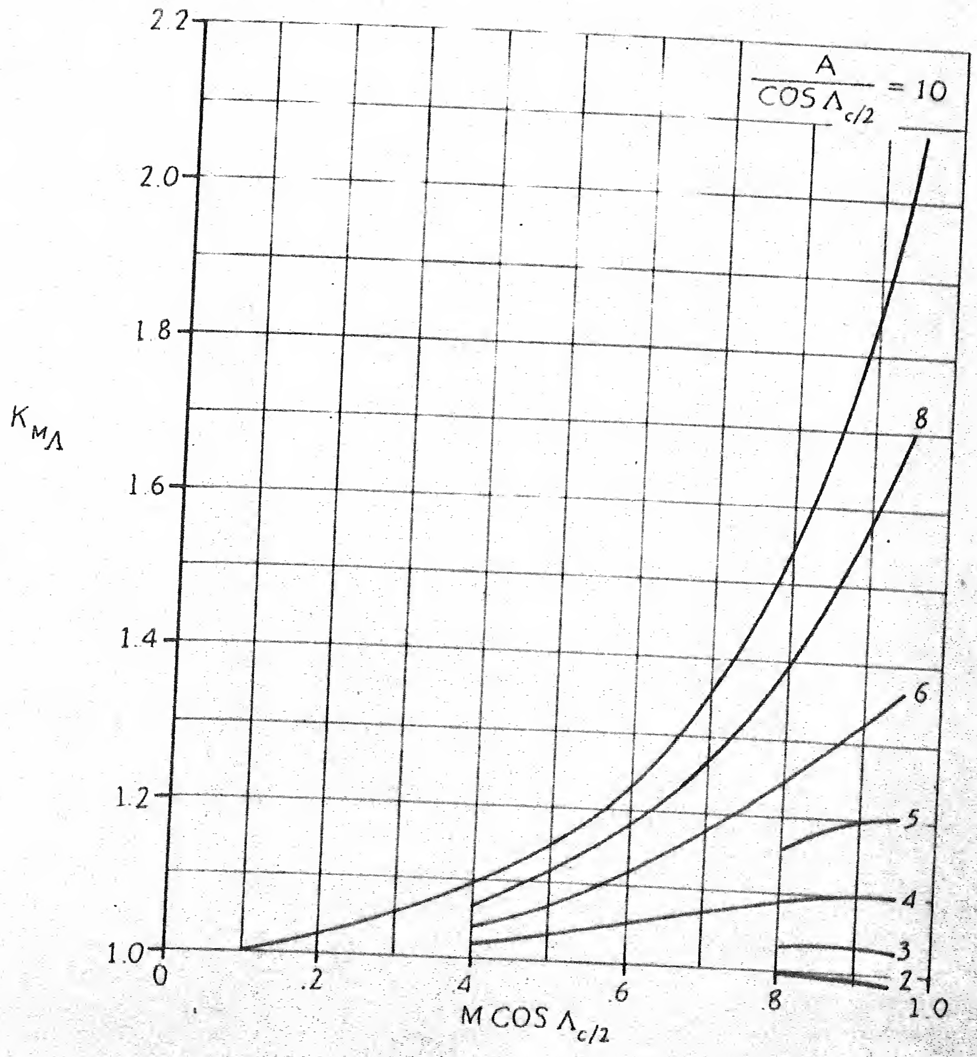


Figure. 12. Compressibility Correction Factor to Sweep Contribution to Wing C_{l_β}

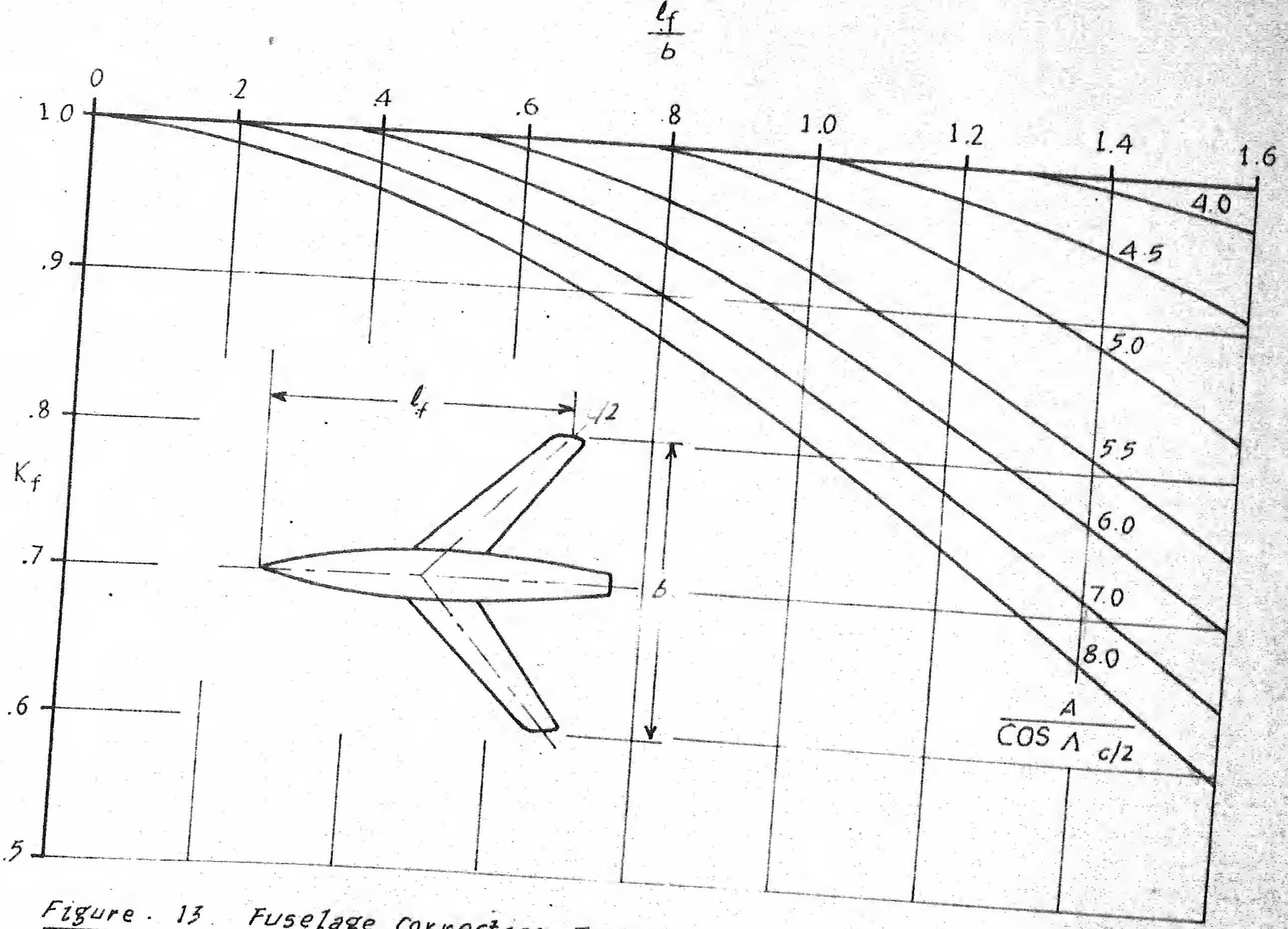


Figure. 13 Fuselage Correction Factor

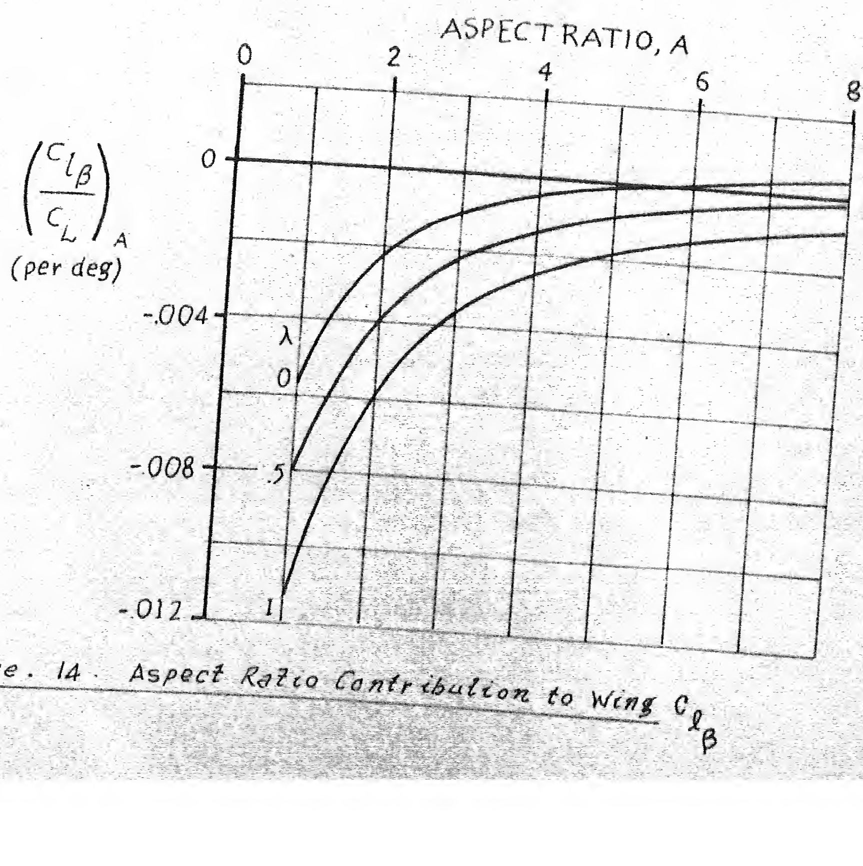


Figure. 14 Aspect Ratio Contribution to Wing C_{l_β}

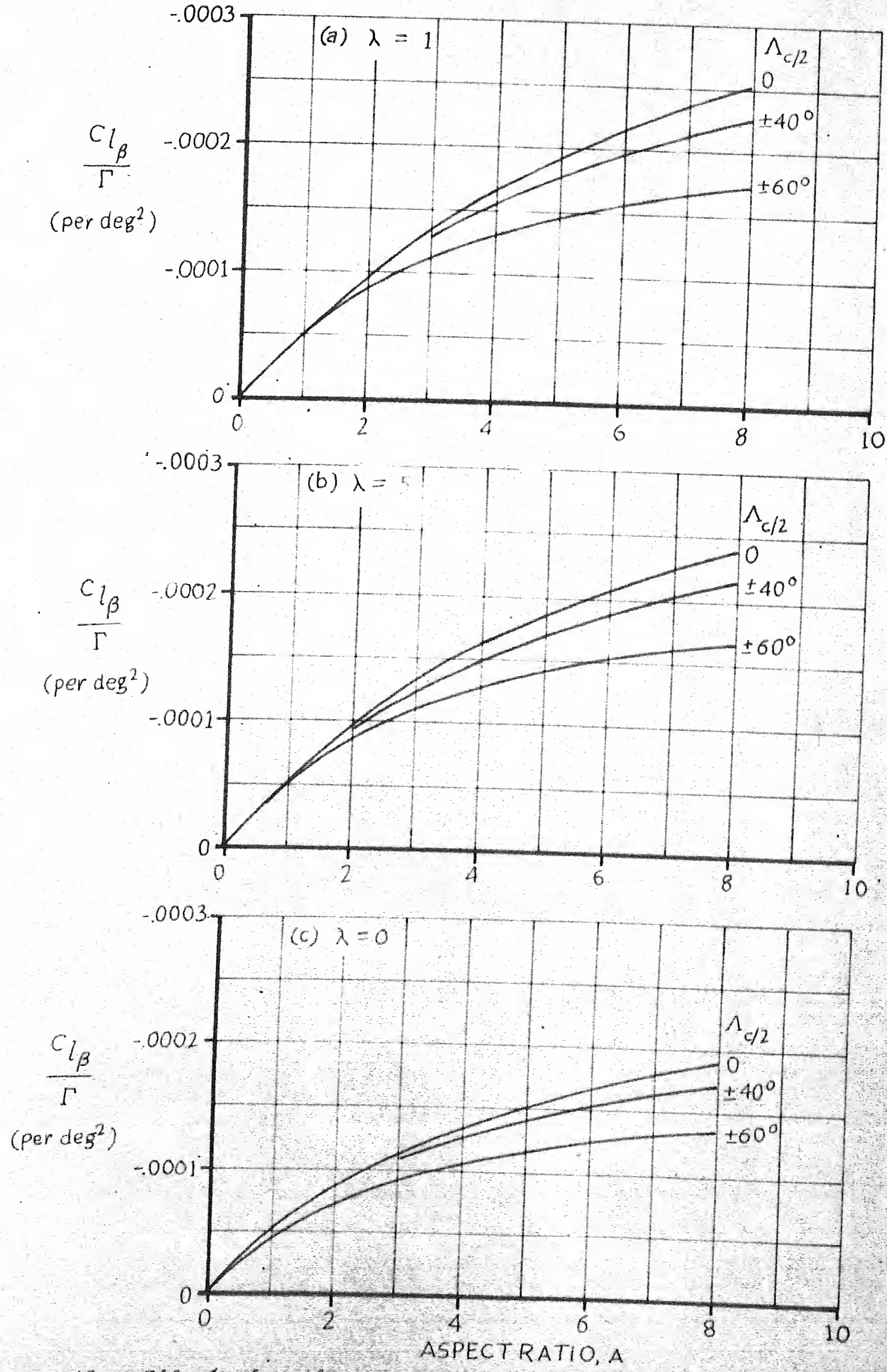


Figure 15. Effect of Uniform Geometric Dihedral on Wing C_{l_β}

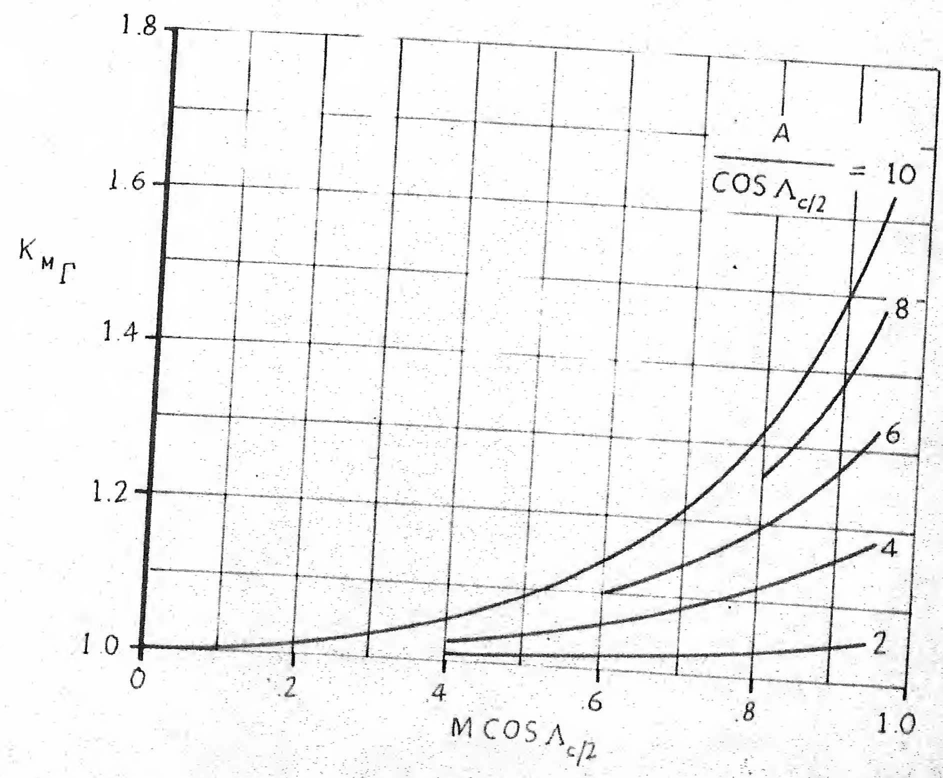


Figure 16. Compressibility Correction to Dihedral Effect on Wing C_{ℓ_β}

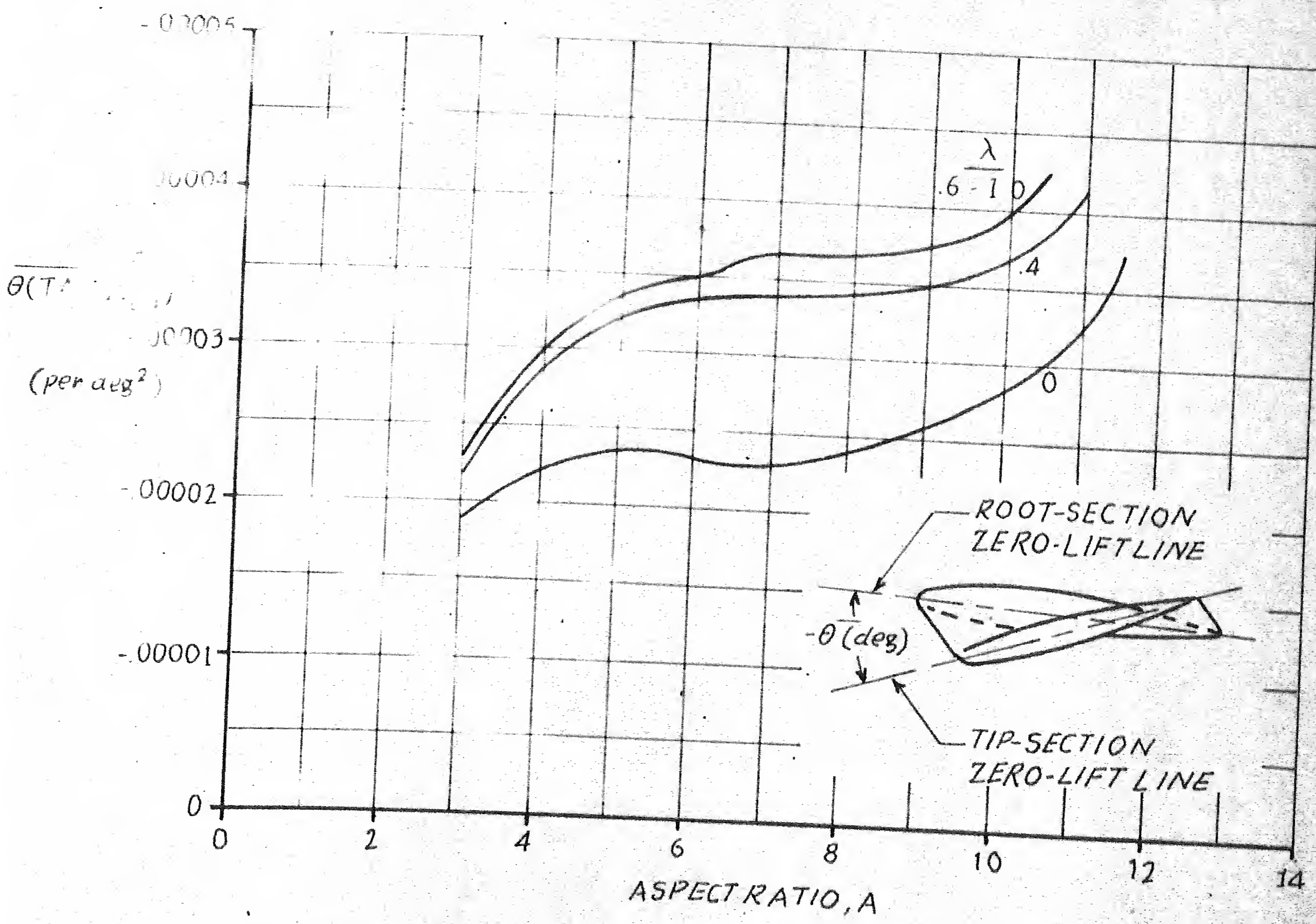


Figure .17. Effect of Wing Twist on Wing C_{l_β}

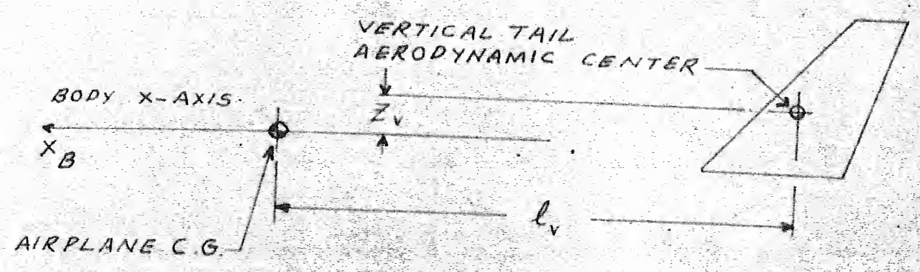


Figure .18. Geometry for Determining the Location of the Vertical Tail Aerodynamic Center in Relation to the Overall Airplane Center of Gravity in Body Axes.

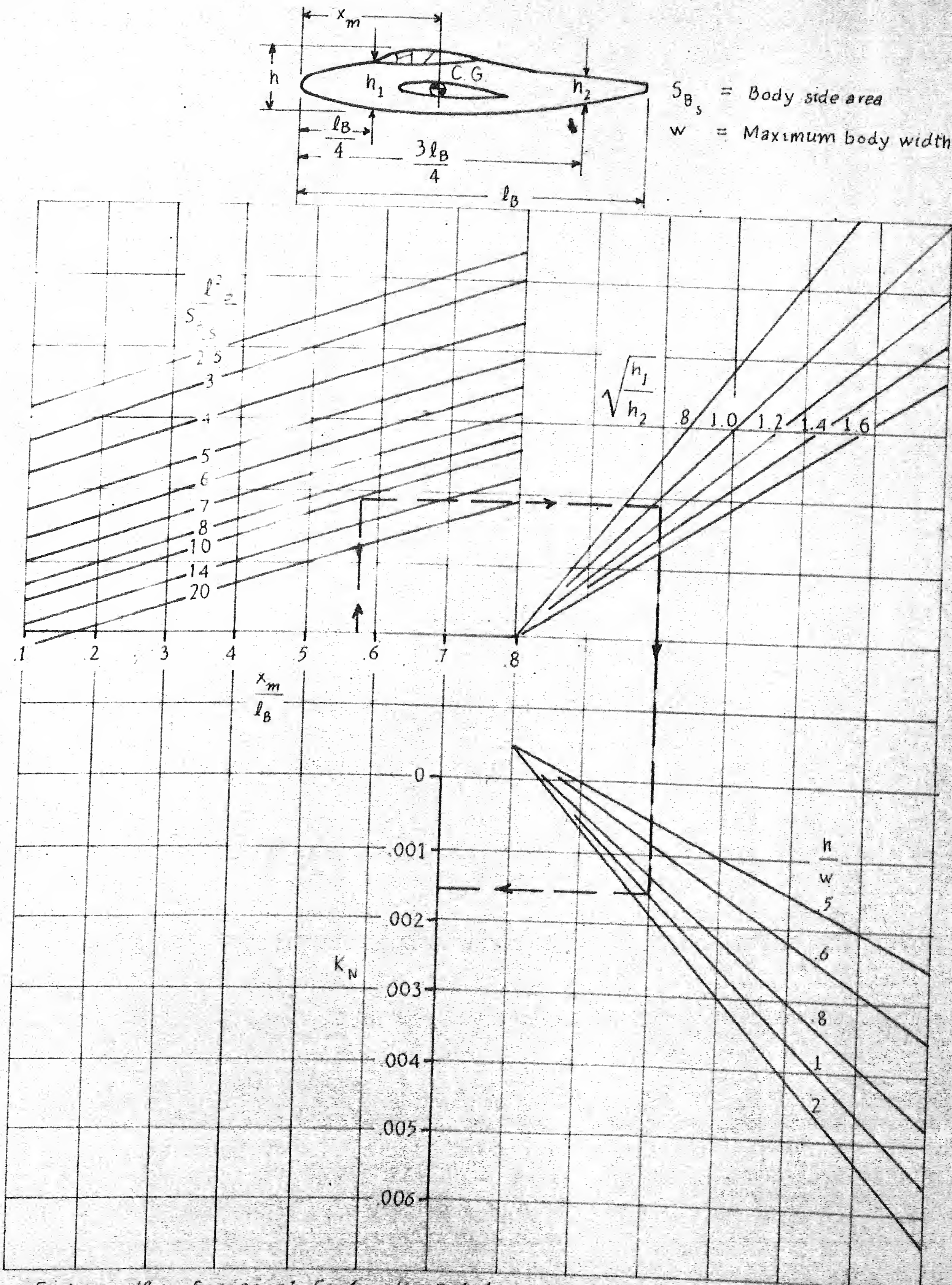


Figure 19. Empirical Factor K_N Related to Sideslip Derivative C_{n_β} for Body + Wing-Body Interference

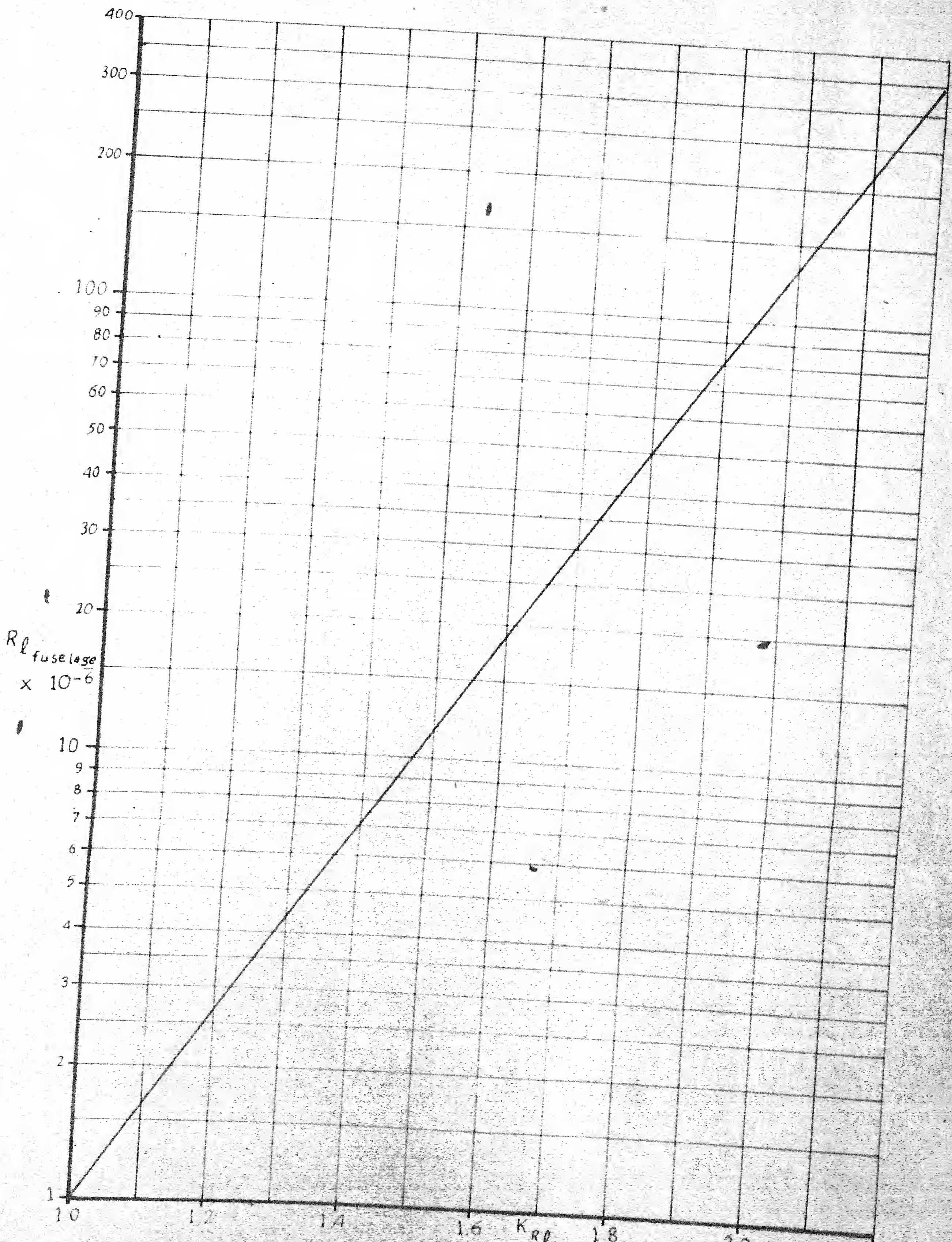


Figure 20. Effect of Fuselage Reynolds Number on Wing-Body C_{n_β}

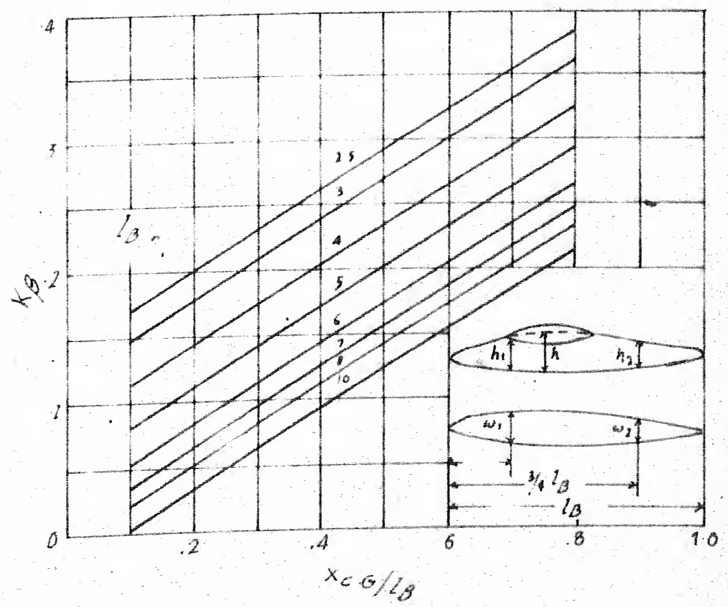


Figure: 21. Fuselage Directional Stability Coefficient

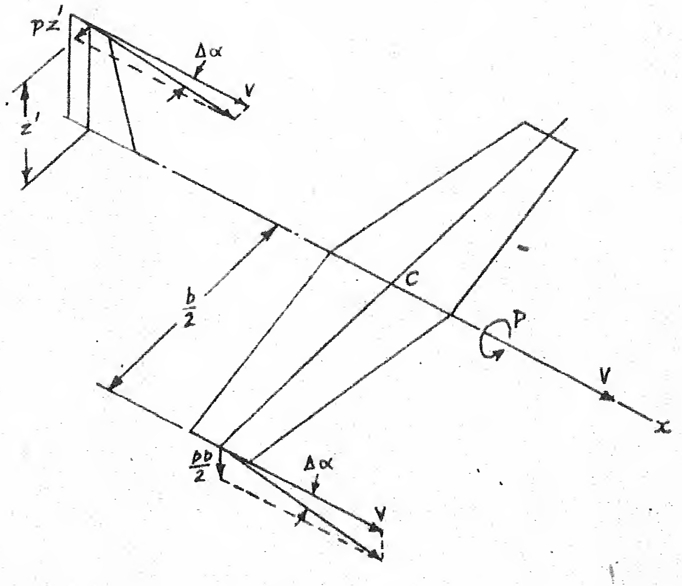


Fig. 22. Angle Of Attack Changes due to p

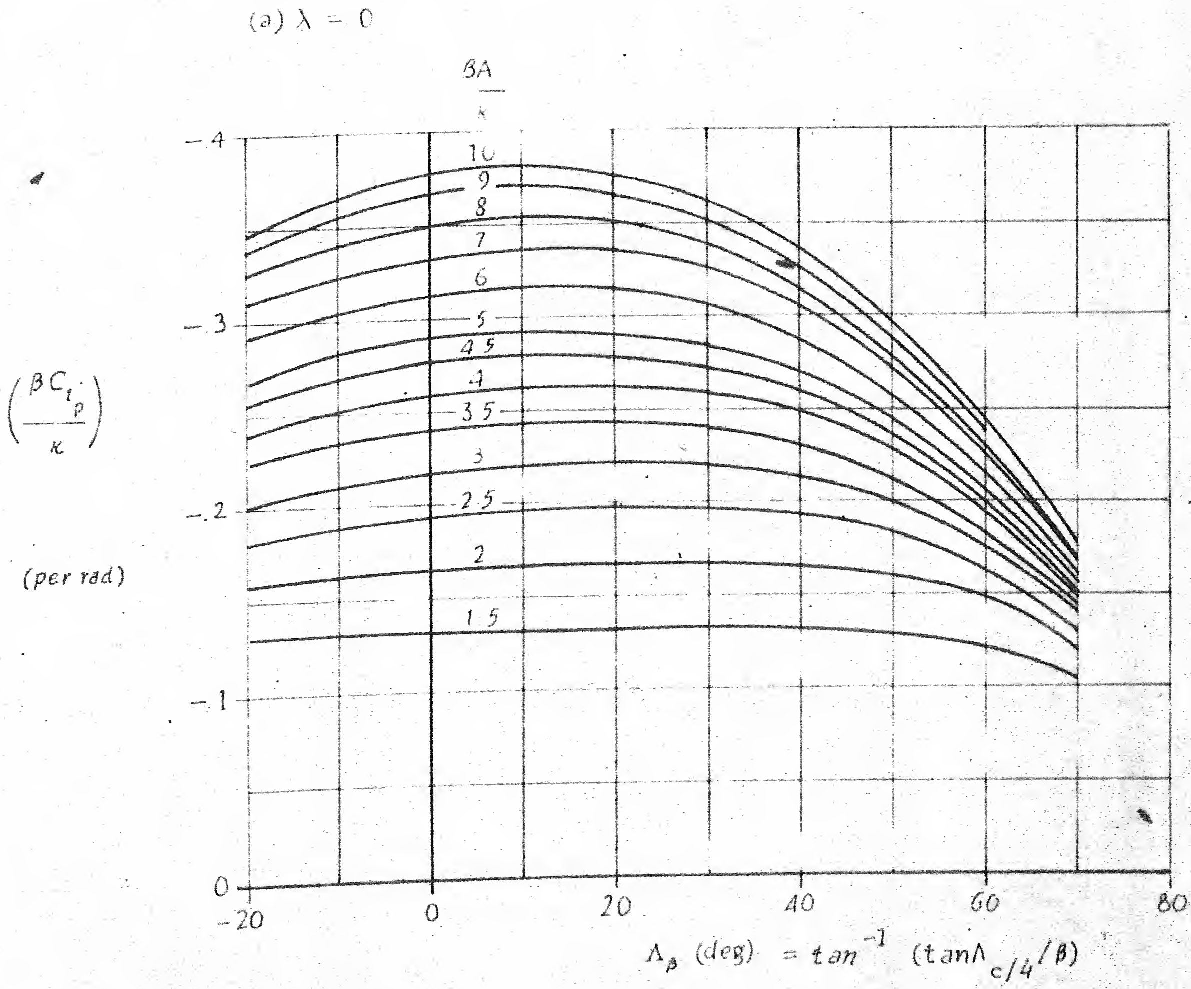


Figure 23a. Roll Damping Parameter

(b) $\lambda = 0.25$

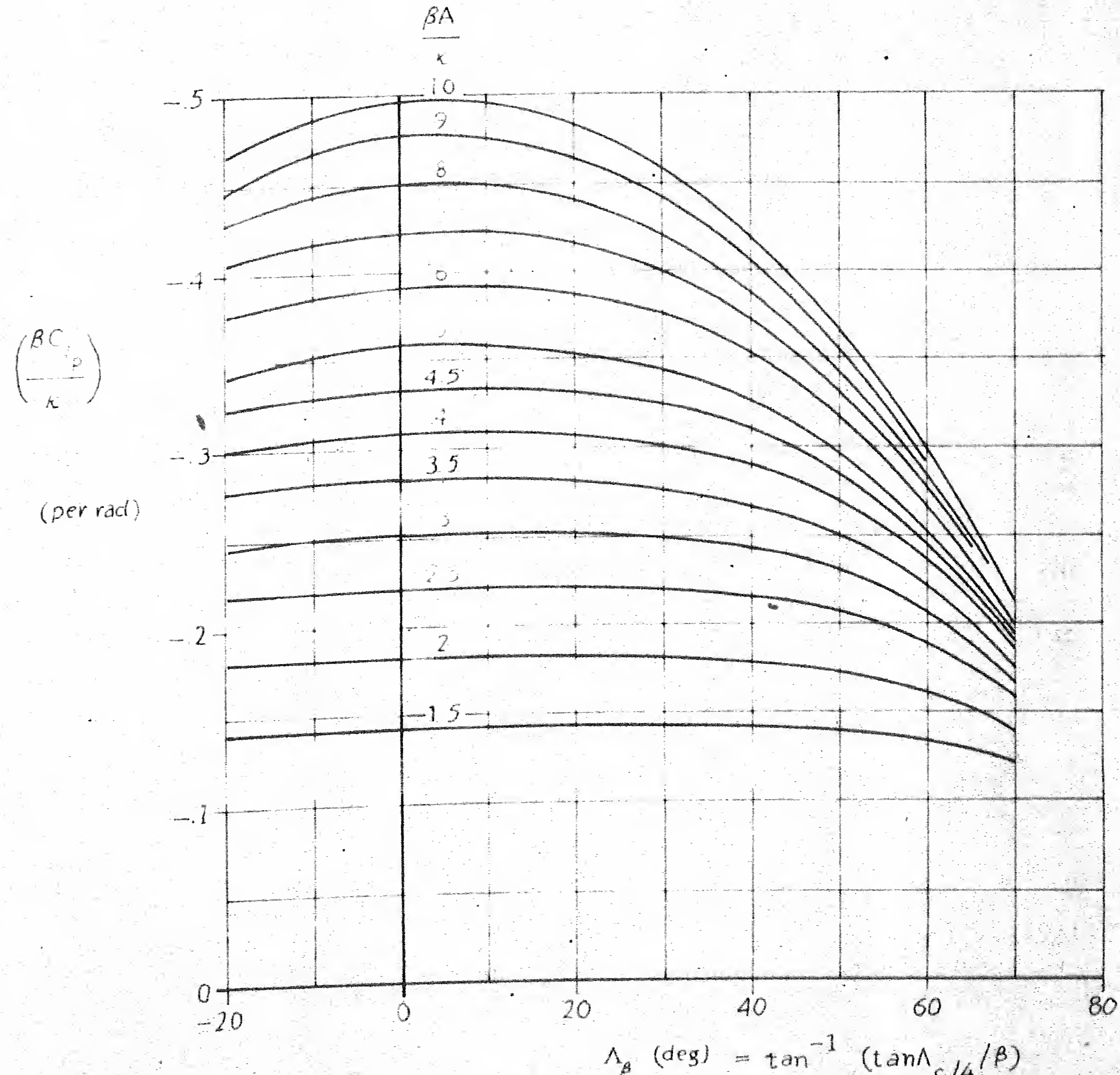


Figure 23b. (Continued) Roll Damping Parameter

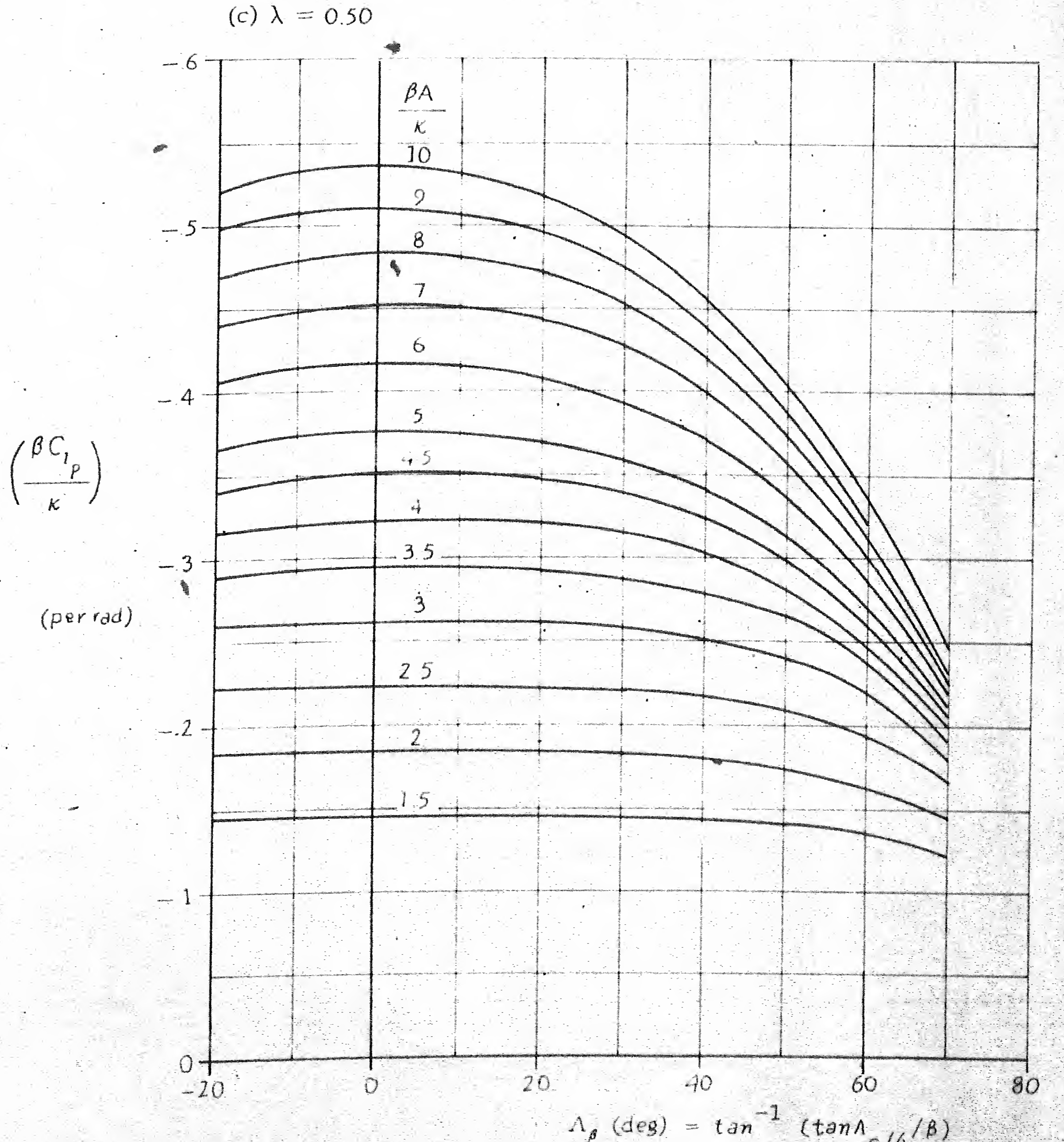


Figure 23C. (Continued) Roll Damping Parameter

(d) $\lambda = 1.0$

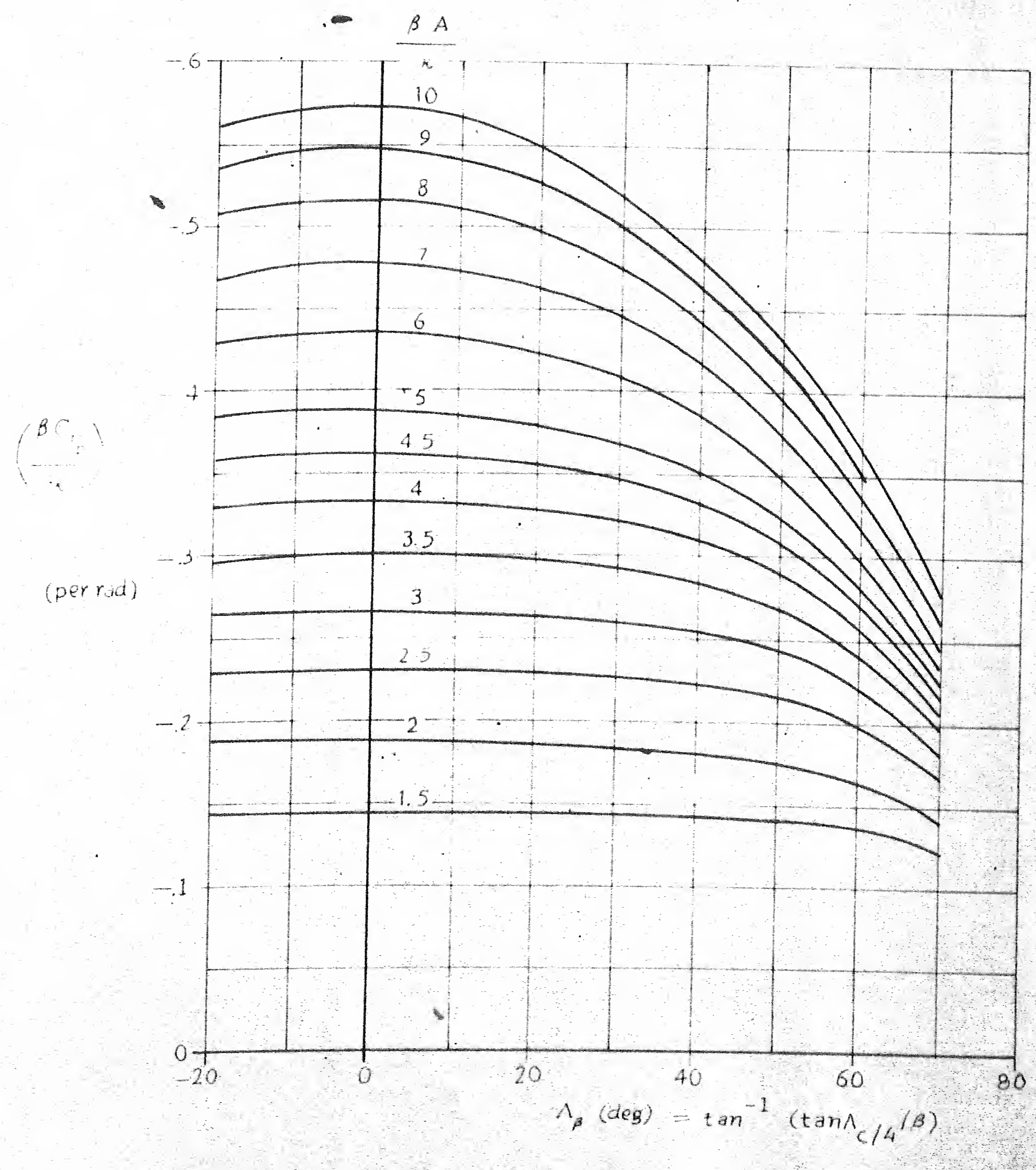


Figure 23d (Continued) Roll Damping Parameter

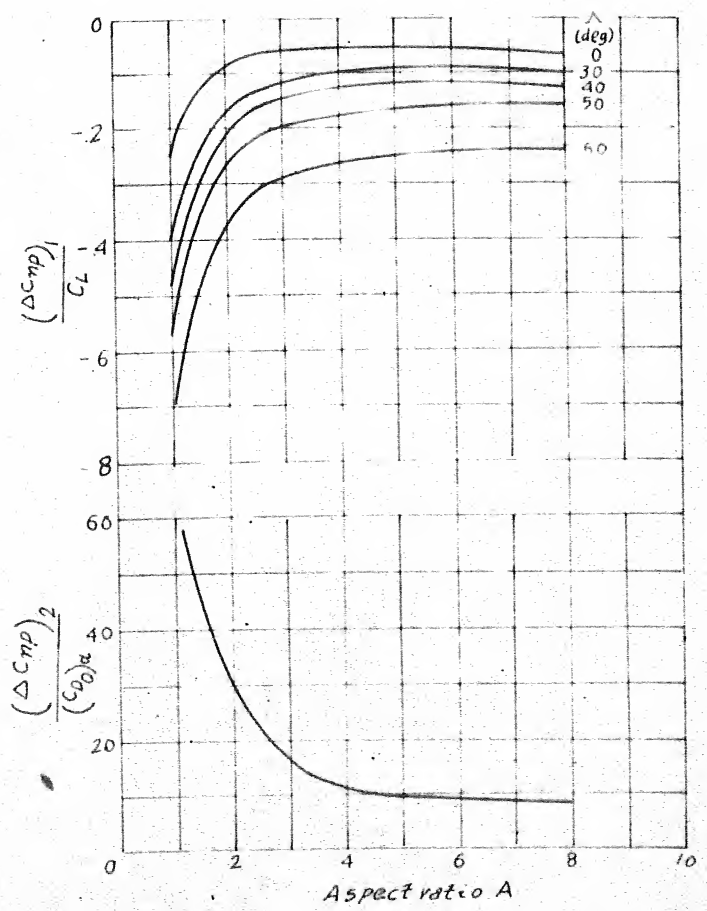


Figure 24. Variation of $\dfrac{(\Delta C_{np})_1}{C_L}$ and $\dfrac{(\Delta C_{np})_2}{(C_{D_0})_\alpha}$ with Aspect Ratio.

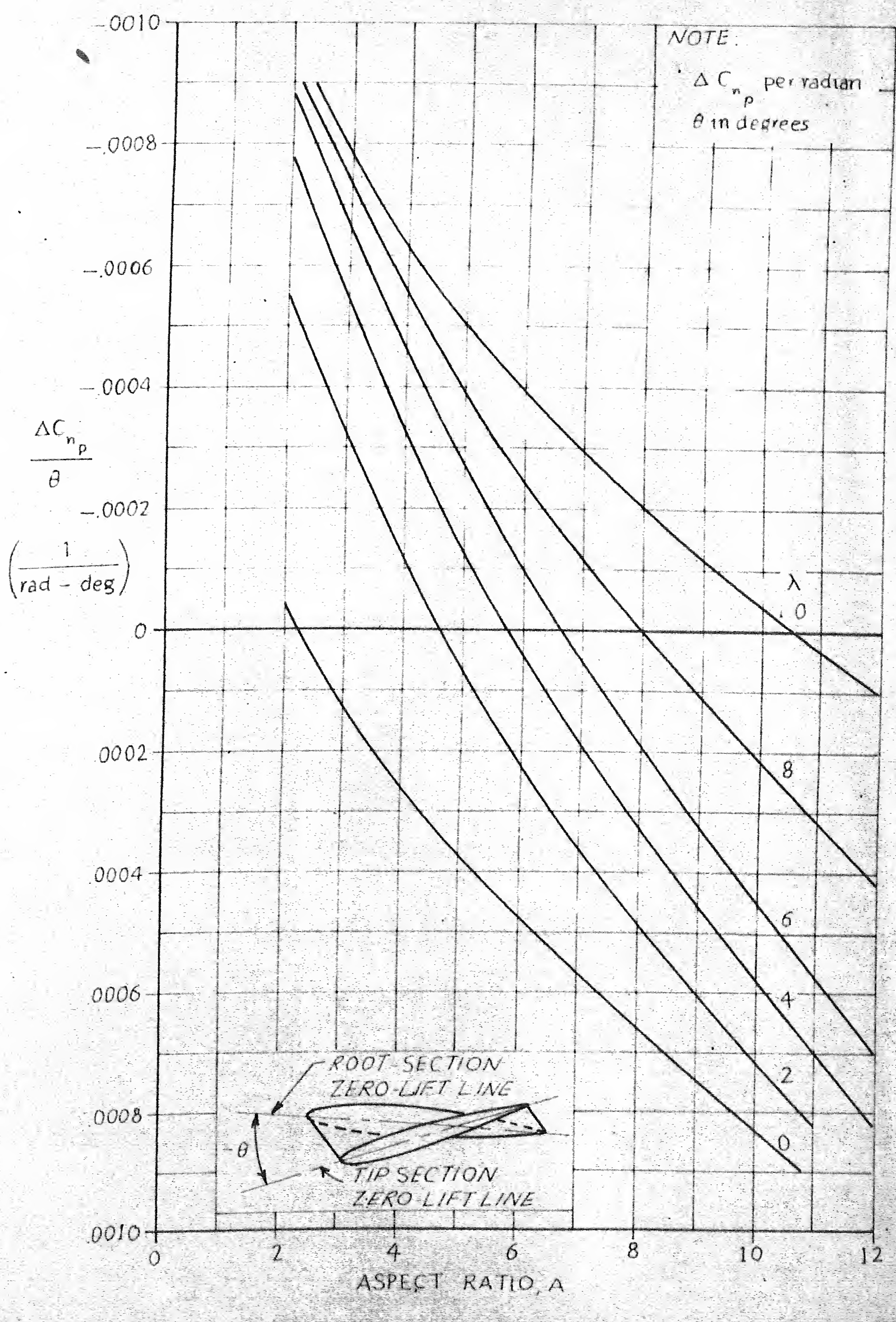


Figure 25. Effect of Wing Twist on Wing Rolling Derivative C_{n_p}

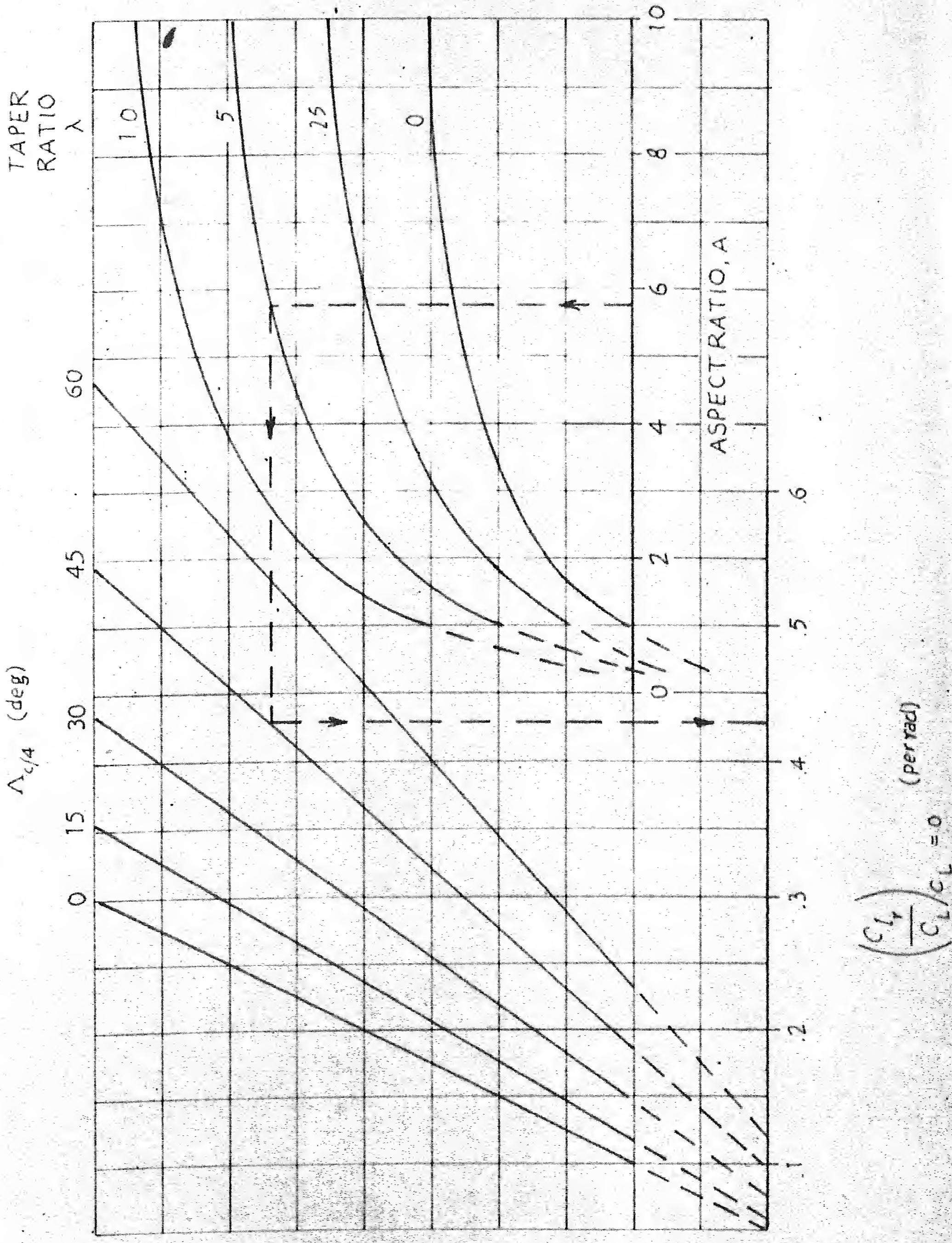


Figure 26. Wing Yawing Derivative C_{ℓ_r}

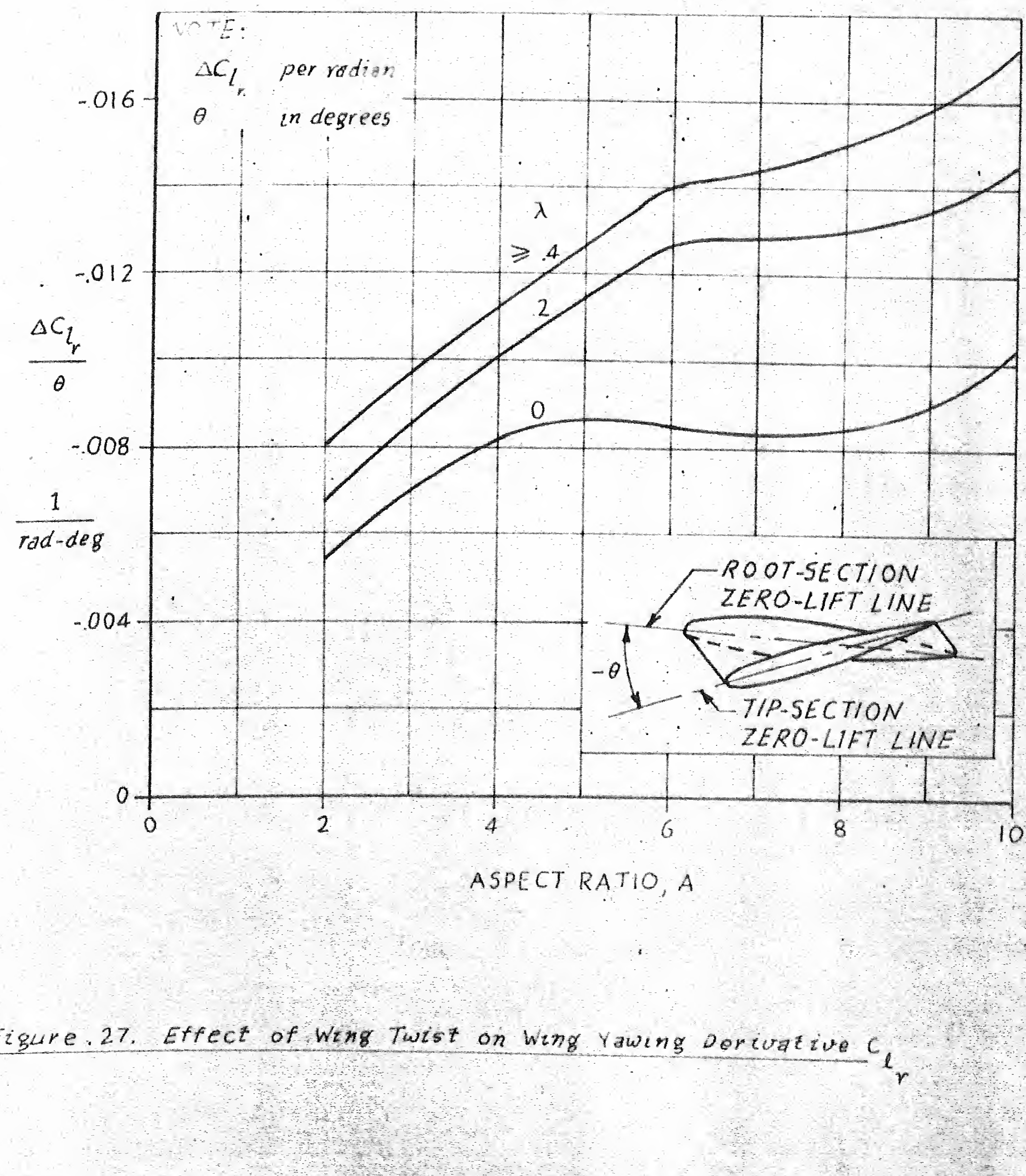


Figure 27. Effect of Wing Twist on Wing Yawing Derivative C_{l_r}

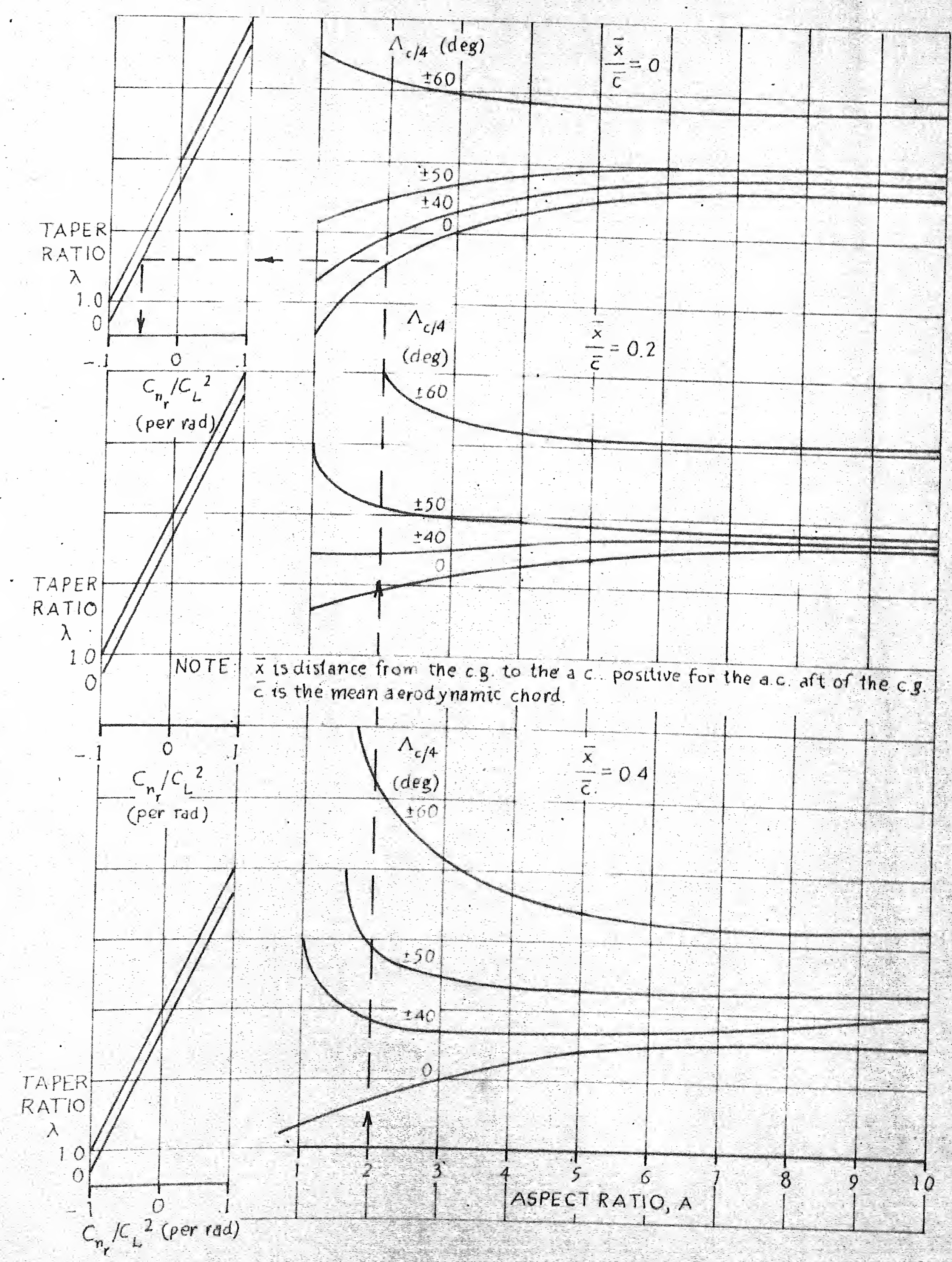
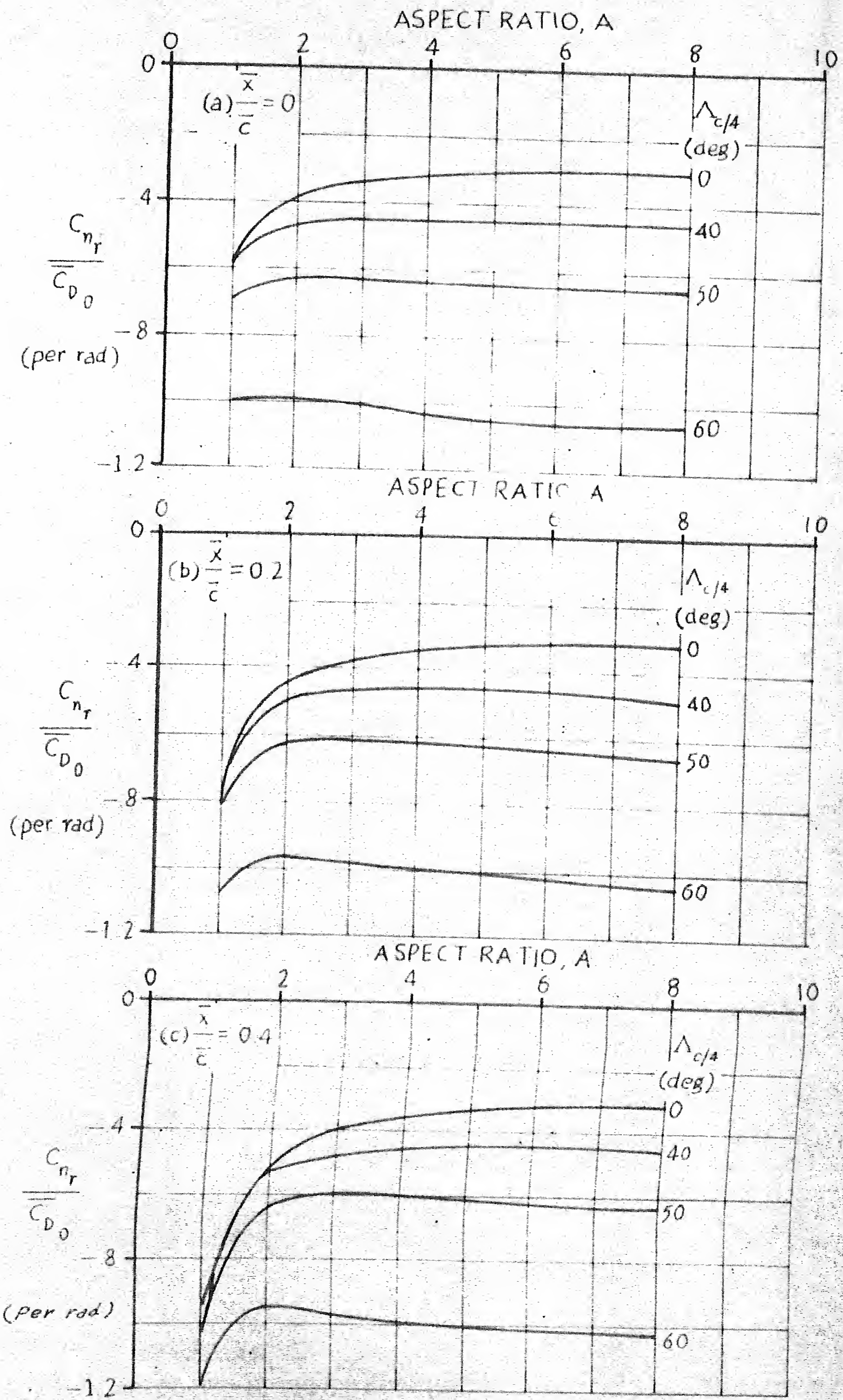


Figure 28. Low Speed Drag-Due-to-Lift Yaw-Damping Parameter



NOTE: $\bar{x}$ is the distance from the c.g. to the a.c., positive for the a.c. aft of the c.g.
$\bar{c}$ is the wing mean aerodynamic chord.

Figure 29. Low Speed Profile-Drag Yaw-Damping Parameter

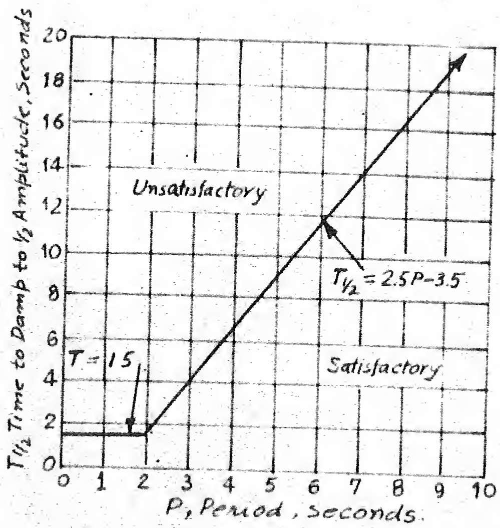


Fig. 30. Required Damping of the Lateral - Directional Oscillation.